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BEHAVIOR OF TURBULENT SCALES IN HYPERSONIC SPHERE WAKES

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BEHAVIOR OF TURBULENT SCALES IN HYPERSONIC SPHERE WAKES

by

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RESUME

On a étudié, à l'aide de sondes électroniques et ioniques, le comportement des macroéchelles associées aux fluctuations des densités de charge au sein du sillage turbulent d'une sphère hypersonique. Des études théoriques ont montré que, dans des conditions favorables, la statistique des fluctuations de courant d'une sonde de Langmuir cylindrique est représentative de la statistique des fluctuations de la densité électronique. Cependant, des études similaires concernant des sondes ioniques ont montré que la statistique des fluctuations de courant n'est pas uniquement fonction des fluctuations de la densité ionique mais est aussi fortement influencée par le couplage des fluctuations de température et de vitesse.

Néanmoins, les mesures que nous avons faites sur les sillages d'une sphère de 2.7 pouces de diamètre, lancée à 14,500 pieds/seconde, dans une atmosphère d'azote à 7.6 et à 20 torr, ou d'air à 10 torr, indiquent que les macroéchelles des deux types de sondes sont comparables. Ce fait accroît grandement la crédibilité des mesures de macroéchelles des sondes ioniques lesquelles sont plus complètes que celles des sondes électroniques.

La valeur moyenne des macroéchelles mesurées au CRDV à l'aide de sondes ioniques décroît pour des distances axiales comprises entre 100 et 1000 diamètres. Toutefois, les résultats des sondes électroniques révèlent que, passé 1000 diamètres, la valeur moyenne se remet à croître. Pour des distances axiales inférieures à 400 diamètres, les résultats des sondes ioniques montrent que la macroéchelle moyenne est fonction du produit $P_\infty D$. Si l'on réunit les mesures faites au CRDV avec celles en provenance d'autres laboratoires, on obtient un comportement général où la macroéchelle moyenne commence par décroître pour ensuite croître au fur et à mesure que la distance axiale augmente. Cette confrontation met par ailleurs en évidence le fait que le comportement de la macroéchelle moyenne dépend de la vitesse de la sphère. (N C)

ABSTRACT

The behavior of macroscales associated with the charge density fluctuations in the turbulent wakes of hypersonic spheres has been investigated using both Langmuir electron probes and continuum electrostatic ion probes. When cylindrical Langmuir probes are operated under favourable conditions, theoretical analysis indicates that the statistics of the probe current fluctuations are representative of the statistics of the electron density fluctuations. Similar analysis for continuum ion probes indicates however that the statistics of the current fluctuations depend not only on the ion density fluctuations but are possibly also strongly influenced by correlation functions involving temperature or velocity fluctuations.

Nevertheless, in the present experiments involving probe measurements in the wakes of 2.7 inch diameter spheres flown at 14,500 feet/second in 10 torr air and in 7.6 and 20 torr nitrogen atmospheres, macroscale measurements with ion probes appear to be equivalent to macroscale measurements from Langmuir probes. This result greatly increases the credence of the ion probe macroscale data which, in fact, are considerably more extensive than those obtained with Langmuir probes.

The mean macroscales obtained at DREV with ion probes decrease in magnitude as axial distance increases between 100 and 1000 diameters; the Langmuir probe results which extend to slightly larger axial distances indicate the mean scale begins to increase again between 1000 and 2000 diameters. For axial distances less than 400 diameters, the ion probe results also indicate a significant dependence of the mean macroscale on the paramter $P_{\infty}D$. When compared with previous macroscale results obtained in sphere wakes by a variety of techniques, the DREV measurements appear to fit into a general pattern in which the mean macroscale first decreases and then increases with increasing axial distance. This same comparison indicates that the mean macroscale behavior is dependent on sphere velocity (U)

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1.0 INTRODUCTION

The fluctuating properties of various physical observables in the turbulent wakes of hypersonic bodies have attracted considerable interest in the past decade (1 - 10). The measurements of the characteristics of the electron density fluctuations has been awaited with particular interest, since the scale sizes of these fluctuations are important for the interpretation of the scattering of microwave radiation by wakes (11 - 13).

Much of our experimental knowledge of hypersonic turbulent wakes has been obtained in ballistic ranges. This type of facility permits the laboratory study of free wakes over axial distances extending to tens of thousands of body diameters behind a projectile. To date, a number of techniques have been applied to obtain experimental estimates of the turbulent scales and spectra associated with the fluctuations of gas density, temperature, velocity, and charge density in the wakes of both spherical and conical projectiles (1 - 10).

One of these techniques, exploited mainly at the MIT Lincoln Laboratory (1 - 5) involved schlieren photography of the wake at preselected axial distances behind the projectile. Photodensitometer traces of the film contrast in the photographs were subsequently obtained along directions parallel and perpendicular to the wake axis. These data were analyzed to obtain the autocorrelation functions of the film contrast fluctuations. Finally, the autocorrelation functions for the gas density fluctuations were estimated using the mathematical relationships which had been established between the autocorrelation functions for gas density fluctuations and those for film contrast fluctuations (2).

The first detailed measurements of film contrast fluctuations at Lincoln Laboratory were made on photographs of the wakes of 3/8 inch diameter spheres flown at 8,000 feet/second (2). The results

obtained were claimed to support the hypothesis that the turbulence in these wakes was statistically homogeneous and isotropic (2). Within the accuracy of the measurements, the autocorrelation functions and the spectra obtained from their transformation were independent of the product of pressure and sphere diameter ($P_\infty D$) and also independent of the axial distance (X/D) behind the projectile as measured in sphere diameters. The correlation length or scale (defined as the distance required for the magnitude of the autocorrelation function to be reduced by a factor of 1/e) had a constant value of approximately 0.5 diameter, independent of position in the wake.

Subsequently, contrast fluctuation measurements were made from the wakes of 0.5, 1.0, and 1.6 centimeter diameter spheres, flown at 18,000 feet/second (5). In agreement with the previous findings at lower velocities, the results indicated that the observed turbulence was statistically homogeneous and isotropic. Similarly, the autocorrelation functions were again found to be independent of the parameter $P_\infty D$. However, there was a substantial difference between the behavior of correlation lengths (or space scales) in the wakes of spheres at 8,000 feet/second and at 18,000 feet/second: the correlation lengths from the low velocity sphere wakes were of the order of 0.5 diameter for all axial distances but in the wakes of 18,000 feet/second spheres, the lengths were constant between 0.2 to 0.3 diameter over an axial distance of 1600 diameters from the body. At larger axial distances they tended to increase approximately as the half power of axial distance.

Most of the other experimental techniques used to observe the fluctuations in turbulent wakes depended on the insertion of various types of probes into the wake. Through the use of such probes, a considerable amount of information was generated concerning the turbulent scale sizes and the frequency spectra associated with the fluctuations of various physical quantities in turbulent sphere and cone

wakes. With some exceptions, most of this work was done with small caliber projectiles and there are few results for the first 200 or 300 diameters of wake just behind the projectile. Much of the data has been contributed by Fox and his coworkers (6 - 10) at TRW. Their early work was done with hot wires and hot-film anemometers in the wakes of several kinds of 0.22 caliber bullets flown at 4000 feet/second (6, 7); later work includes results obtained with both hot-film and cooled film anemometers in the wakes of 0.5 inch diameter spheres and 0.38 inch slender cones flown at 6,000 and 6,500 feet/second respectively (8). The interpretation of the anemometer signals is not a strong point of these measurements: the sensors are operated at constant temperature and the heat exchange with the environment depends on both the local flow velocity in the environment and on the difference of the local environment temperature and the sensor temperature. Fluctuations in the sensor signal are therefore caused both by velocity and temperature fluctuations in the environment. To sidestep this difficulty, the following expedient was employed: anemometer signals were interpreted as being 'dominated' by fluctuations in wake temperature or velocity, according to the magnitude of the difference in temperature between the sensor and the wake. If this difference is small, the anemometers are mainly sensitive to temperature fluctuations; if the difference is large, the velocity fluctuations are contributing significantly to the fluctuations in the anemometer outputs.

The estimates obtained at TRW for turbulent scales dominated by temperature fluctuations extend over a range of axial distance from 300 to 1000 diameters, in the case of the wake of spheres. The space scales for temperature fluctuations vary between 0.5 and 1.0 diameter for 6,000 feet/second sphere wakes. The TRW anemometer scale data, which are influenced significantly by velocity fluctuations, extend over a larger region of the wake - from about 300 to 10,000 diameters behind the projectile. Between 300 and 2000 diameters, the value of the space scale for velocity fluctuations remains constant at about 0.4 diameter;

however, after 2000 diameters, the velocity scale increases with axial distance, reaching a value of about 0.8 diameter at an axial distance of 10,000 diameters. In the region where both temperature and velocity scale estimates exist, the TRW results indicate that the velocity eddies are smaller than the temperature eddies (8).

The highest projectile velocities used in the TRW experiments are roughly just on the borderline of being hypersonic. However, recently Fox and Rungaldier (9) achieved Mach 9 by flying 0.5 inch diameter spheres at 6,000 feet/second in an atmosphere of monatonic krypton at 20 torr. Sufficient ionization (up to 10^{10} electrons/cubic centimeter) was obtained to permit the study of electron density fluctuations over an axial distance of 200 to 800 sphere diameters. The indicated magnitude of the space scales for electron density fluctuations was about 0.3 diameter, independent of axial distance in this region of the wake. Compared with the previous TRW anemometer scale data, the electron density scales are of the same order of magnitude and perhaps just slightly smaller than the velocity space scales.

While some information already exists on the behavior of the scale sizes associated with the fluctuations of various quantities in a turbulent wake, more measurements are obviously needed. Most of the existing data have been obtained at normalized radial distances that are large compared to unity and little information is available from the region of the wake just behind the projectile. Of particular interest would be systematic measurement of the behavior of scale size as a function of radial and axial distances in the wake. Knowledge of the intermittency structure of the wake in the region where scale measurements are made is also important.

Measurements of the scale sizes associated with ion and electron density fluctuations have recently been attempted at the Defence Research Establishment Valcartier (DREV). In one experiment, observations were effected with a transverse array containing up to eight ion probes situated across the wake; in the other, observations were made with an

axial array of electron collecting Langmuir probes. The experiments were made on the wakes of large 2.7 inch diameter spheres flown at approximately 14,500 feet/second. The projectiles were roughly five times as large as the 0.5 inch spheres whose wakes were studied by Fox and his coworkers, and consequently offered significant advantages in terms of both spatial and temporal resolution. These advantages were particularly useful for the study of the near wake, where conditions may change rapidly (14 - 20). Finally, this research complements other studies of hypersonic wakes at DREV under similar conditions, including investigation of the mean velocity distribution (14, 15) the mass density distribution (16), the mean charge density distribution (17, 18), and the wake structure (19, 20).

The following report is an account of the information which can be deduced with ion and Langmuir probes concerning the charge density fluctuation scales in a hypersonic turbulent wake.

2.0 DIFFICULTIES WITH TURBULENCE MEASUREMENTS IN WAKES USING CHARGED PARTICLE PROBES

There are two types of problems encountered when charged particle probes are used to observe the electron or ion density behavior in free turbulent hypersonic wakes. One inherent difficulty, particularly with ion probes, is in the interpretation of fluctuating probe currents. The other problem arises because the wake in a ballistic range is moving with respect to the laboratory coordinates while the probes are fixed in the laboratory.

2.1 Fixed Probes in Moving Wakes

The best way to describe the difficulty that arises when an attempt is made to estimate turbulent scales and spectra through observations made by a fixed probe on a moving wake, is to give an example:

The schlieren technique was mentioned in the Introduction to this report as a means of photographing the total effect of density fluctuations in the wake. The pattern of turbulence represented on a schlieren photograph is frozen at the instant when the film is exposed, and all observations or measurements subsequently made on the film contrast correspond to that same instant of time. Let us contrast this type of observation to those which depend on the insertion of various types of probes into the wake. In the probe techniques, one depends on the mean velocity of the wake to transport the flow carrying the detected physical quantity past the sampling probe. The signal collected by such a probe cannot represent observations made at the same instant of time; instead the signal represents a time history of a fluctuating pattern transported past the probe during a certain time interval. If this turbulent pattern were frozen in time and the mean wake velocity constant, this signal time history would directly correspond to the spatial variation in the wake of the detected fluctuating physical quantity. In practice the turbulent pattern is not frozen, but is varying in time (21); the signal time history represents a mixture of both the temporal and the spatial variation of the pattern (21). Fortunately, measurements in hypersonic wakes have shown that the time scale of the turbulence can often be large compared to the time required by the flow velocity to convect the wake past one or more probes (22).

The coordinate system with respect to which wake measurements are made is also confusing: in contrast to measurements made with hot wires in turbulent jets where the probe remains fixed at a certain distance from the jet until moved by the experimenter, the probe in the free wake of a projectile flown in a ballistic range continually changes position with respect to the projectile as the latter recedes down the range. (In other words, the probe wake measurements are referred to a range-fixed coordinate system rather than to a projectile-fixed coordinate system.) Additionally the mean flow velocity varies with the axial distance of a probe behind the projectile and the flow past the probe is subject to velocity fluctuations. All of these effects tend to introduce distortions into the space scales and spectra of the fluctuating

variables which are deduced from analysis of the autocorrelations of the recorded probe signals. The effect on the turbulent spectra will occur mainly at low wave numbers (because of nonuniformity in convection velocity) and at intermediate wave numbers (because of temporal variation). In both cases the condition is aggravated because the observations are made in range-fixed coordinates instead of projectile-fixed coordinates.

When measuring scale sizes, many of the difficulties described above can be circumvented by using pairs of probes at various distances of separation. By time-averaging over the signals from pairs of probes it is possible to obtain estimate of the values of the space correlation function at a number of preselected points on the correlation function. (Care must be taken to avoid problems of aerodynamic interference between the probes produced when the interprobe spacing is made too small (23).) In practice, the signals from probes immersed in the wake are nonstationary and must be divided into a number of consecutive segments (14) over the duration of which they may be considered as being at least quasi-stationary. Unfortunately, the total duration of the signal from most wake probes is measured in milliseconds, and the time-averaging which is achieved over a segment of such a signal is quite inadequate to define the shape of the correlation function with statistical precision (24). However, it is still possible to obtain reasonable estimates of the scale sizes associated with the correlation functions.

2.2 Experimental Techniques with Electron and Ion Probes

The experimental techniques with which electron probes and ion probes were exploited at DREV to estimate the mean characteristics of the charge density distributions in the wakes of hypersonic spheres have been described in detail in recent publications (17, 18). Only a brief description will be given here.

In the case of the electron probes, four fine wire (Langmuir) probes were placed one behind the other to form an axial array parallel to the line of flight. This geometry lends itself to the measurement of space correlation information in the direction parallel to the wake axis. The individual cylindrical probes were from 2 to 4 millimeters in length and were each oriented perpendicular to the wake axis; the interprobe separation was 0.5 inch and the diameter of the platinum wire used in the probes varied from 0.0005 to 0.001 inch. At the low pressures employed, the electron-neutral mean free path in the wake was greater than the probe diameter, so that the probes could be considered as collecting electrons under free-molecular conditions (18).

With the ion probes, a transverse survey array consisting of eight two-probe elements, each separated by 1½ inch, was used to obtain a simultaneous sampling of the ionization across the wake (17). In each basic element of the survey array, there were two cylindrical ion probes, one mounted behind the other so as to form an axial pair capable of measuring space correlation information in the direction parallel to the wake axis. A transverse array of such two-probe elements can provide a simultaneous sampling across the wake of space correlations in the axial direction. The probes were 2 millimeter long cylinders fashioned from 0.011 inch diameter gold wire, and the separation of the two probes in an element was 0.375 inch. Using probes of this geometry, analysis based on certain non-dimensionalized parameters evaluated under range operating conditions indicated collision-dominated ion motion, with thick sheaths predominating and a strong convection component in the incident ion flux (17).

The same type of current-to-voltage operational preamplifiers were used with both the electron probes and the ion probes; in all cases the feedback resistor was 200 kilohms. These preamplifiers also supplied the bias voltage applied to the probes with respect to ground. The bias voltage was subsequently stripped by an operational amplifier subtracter circuit and the net voltage signal from the probe transmitted by a

power-amplifier over a terminated coaxial line to the main recording room. Here the signals from adjacent pairs of probes were displayed together on oscilloscopes and simultaneously recorded on 35 millimeter film using Fastax cameras. Subsequently the filmed signals were digitized by a computer-driven film reader (14, 17).

2.3 Interpretation of Fluctuating Electron Probe Currents

The ambient conditions for the electron probe experiments were chosen to favour the free-molecular orbital motion limited operation of the probes. The range pressure was restricted to 10 torr, and the diameter of the probes was limited to 0.001 inch or less, so that the electron-neutral mean free path in the wake would be greater than the probe diameter (18). The ionization was relatively weak (10^{10} - 10^7 electrons/cubic centimeter) in the wakes of the 14,500 feet/second spheres flown in these experiments, except in the region of the wake very near the body. As a consequence the Debye lengths were correspondingly large, and the ratio of the radius of the sheath about the probe to the radius of the probe was considerably greater than unity. Under these conditions, and with the probe strongly biased, the current i to a cylindrical electron probe varies as

$$i_k \sim n_e (V_k - V_p)^{\frac{1}{2}} \qquad (1)$$

where n_e is the local electron density and
$V_k - V_p$ is the difference between the probe potential V_k and the local plasma potention V_p.

The symbol k denotes the existence of more than one probe.

The technique of diagnosing turbulent plasmas with Langmuir probes has recently been evaluated by Demetriades and Doughman (25). Their findings indicated that under conditions of collisionless electron motion in the sheath, large values of the ratio of sheath radius to probe radius, and a large value for the non-dimensionalized potential,

the electron density and rms electron density fluctuation magnitude can be determined simply with a cylindrical electron probe. No corrections are needed to account for correlations of the other fluctuating variables present in a turbulent wake. The average of the fluctuating current to a cylindrical probe is equal to the probe current obtained at the average plasma properties, while the ratio of the rms current fluctuation to the average current is equal to the ratio of the true rms electron density fluctuation to the mean electron density (25).

The system of electrodes employed in the Langmuir probe equipment consisted of a few small positively-biased probes and a much larger grounded grid consisting of many turns of platinum wire similar to that used for the probes (18). The purpose of the grid was not only to furnish the return path for the current drawn from the plasma but also to hold the plasma potential in the wake near ground potential. In fact, what appeared to happen in practice was that the plasma potential tended to follow the potential applied to the probes. Although in some cases 9 volts with respect to ground was applied to the probes, the plasma potential appeared to adjust so as to reduce the magnitude of $(V_k - V_p)$ to about the order of 1 volt. Even so, the probes could still be considered as having been fairly strongly biased, although not quite as strongly as planned (18).

From Equation 1 it is apparent that if V_k is much larger than V_p, then fluctuations in V_p of the order of V_p have little effect, and all fluctuations in the current i can be attributed to fluctuations in the electron density n_e. However, if V_p is of the same order as V_k, it is possible that fluctuations in the probe current could be caused by fluctuations in the plasma potential.

As previously reported (18), the Langmuir probe equipment was employed to derive simultaneous estimates of the behavior of the electron density and plasma potential by using simultaneous measurements

of current to two probes at different potentials. Unfortunately the two probes were not located at the same point in the fluid but were separated by about 0.5 inch. Thus the values of n_e and V_p which were derived by applying Equation 1 to the current measurements cannot be strictly attributed to a specific point in the fluid. In fact the two probes are probably but rarely in the same particle of ionized fluid, so that each sees a different level of ionization. The analysis technique, however, requires a single estimate to be derived for the electron density, and this must be reflected in the estimates predicted for the plasma potential as a scattering of the data points. This artificial scatter will mask or amplify any scatter that would otherwise be present.

Examination of the deduced behavior of the plasma potential by the described procedure does indicate the presence of some scattering of the estimates (18), however in some cases this is small compared to the apparent difference between the probe and plasma potentials (Figure 1).

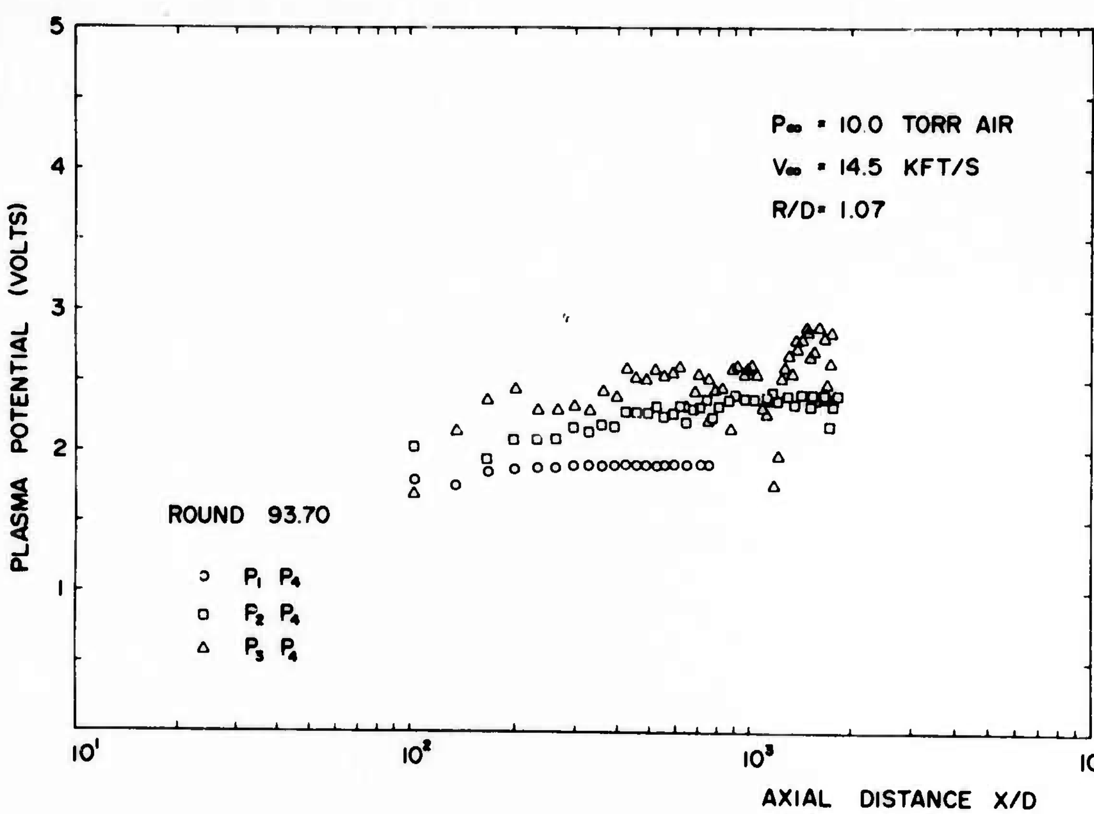

FIGURE 1

Since the analysis procedure in fact forces a certain degree of scattering, there are reasonable grounds for believing that the actual fluctuations in the plasma potential are probably insignificant, although additional work to confirm this would be useful.

2.4 Interpretation of Fluctuating Ion Probe Signals

A great number of theories have been developed to predict the saturated ion current that will be collected by an electrostatic probe immersed in a flowing continuum plasma. They generally agree that the probe current will depend on charge density in the probe vicinity and on other variables such as ion and electron temperature, the pressure, and flow velocity. However almost every theory differs as to the manner in which these additional wake variables affect the probe current.

Among these, the 'static' theories assume the probe current is dominated by diffusion of ions, while the 'kinetic' theories attempt to account for the contribution to the probe current of ions convected by the mean flow velocity. An examination of the dimensionless parameters associated with the various probe theories (17, 26), indicated that for the ambient conditions in sphere wakes and for ion probes of the dimensions employed in these experiments, kinetic theories were required to describe the current collection. However, the kinetic theories were not strictly applicable, particularly in the near wake, because the viscous flow about the probes was not completely immersed in the sheaths (17) and the length to diameter ratio of the probes was smaller than desirable. Interestingly, an experiment involving the observation of the dependence of the probe current on probe orientation gave results which were apparently more in accordance with static theory than with kinetic theory (17). As a consequence of the foregoing considerations concerning ion probe theory, Sévigny et al. (17) attempted to deduce the ion distribution across the wake of a hypersonic sphere by applying three static theories and an equal number of kinetic theories to the current distributions experimentally determined with the transverse survey array. The required data concerning the distributions of velocity and temperature in the wake were obtained from the experimental measurements of Sévigny et al. (14) and

of Dionne and Tardif (16), and by assuming that the required ion and electron temperatures were equal to the neutral gas temperature. Because of the high temperatures existing in the wake, the mobility coefficient μ_i was assumed to vary directly as the square root of temperature and inversely as the pressure. In order to provide a basis for comparison, the electron density distribution in the wake was simulteneously and independently measured by means of a dual channel focussed beam microwave interferometer, operating at X_s band. Since the ion probe measurements were performed in nitrogen atmospheres, oxygen attachment of electrons in the wake to form negative ions was assumed to be minimized.

The results indicated that all the theories overpredicted the ion population in the wake. At 7.6 torr the kinetic theory predictions were from 2 to 5 times higher than the microwave results while at 20 torr their predictions were from 7 to 15 times too high. The predictions of the static theories were about 10 times greater than the kinetic theory predictions. On the basis of their ability to predict the mean ion density under wake conditions, the kinetic theories are somewhat favoured by these results.

An analysis of the usefulness of a number of probe theories for the interpretation of fluctuating probe current in terms of fluctuations in charge density has recently been carried out by Cantin (26). The static theories of Zakharova et al. (27), Su and Kiel (28), Schulz and Brown (29), and the kinetic theory of Kulgein (30) were all investigated. In each case the dependence of the probe current on the different variables in the wake was expressed in power law form. As a result Cantin found that:

$$I_i \sim N_i\, T^{1.5}\, p^{-1} \qquad \text{(Zakharova et al. - Su and Kiel)}$$

$$I_i \sim N_i^{0.8}\, T^{1.3}\, p^{-1}\, V_i^{0.4} \qquad \text{(Schulz and Brown)} \qquad (2)$$

$$I_i \sim N_i^{0.8}\, T^{0.1}\, p^{-0.2}\, V_i^{0.4}\, U_w^{0.8} \qquad \text{(Kulgein)}$$

where I_i represents the ion current
N_i the ion density
T the temperature
p the pressure
V_i the probe bias potential
and U_w the local wake velocity.

The first two expressions are the predictions of static theories, while the third relation describes a kinetic theory. To these may be added the predictions of the theory of Clements and Smy for thin sheaths (31) and the theory of Clements and Smy for thick sheaths (32):

$$I_i \sim N_i^{0.75}\, T^{0.125}\, p^{-0.25}\, V_i^{0.5}\, U_w^{0.75} \quad \text{(Thin Sheath)}$$

(3)

and

$$I_i \sim N_i^{0.67}\, T^{0.17}\, p^{-0.33}\, V_i^{0.67}\, U_w^{0.67} \quad \text{(Thick Sheath)}$$

The last expression is approximated from the true equation of Clements and Smy by neglecting a term proportional to $|\ell n\, (I_i/n_i U_w)|^{2/3}$. (In deriving these expressions from the various theories, whenever the mobility coefficient μ_i appeared, its dependence on temperature and pressure has been approximated as $\mu_i \sim \mu_o\, T^{\frac{1}{2}}\, p^{-1}$.)

The predicted dependence of the rms probe current fluctuation on the rms values of the various wake variables and their cross-correlations can be determined by writing each quantity as the sum of a mean quantity and a fluctuating component (i.e. $I_i = \overline{I}_i + \Delta I_i$), performing a first order expansion, squaring and finally averaging in time. Since the flow in the wake is subsonic, pressure fluctuations can be neglected (33). Additionally the ion probes are strongly biased, so the effects of potential fluctuations can be neglected. These assumptions lead to the following results:

$$\frac{\overline{(\Delta I_i)^2}}{I_{io}^2} = 0.64\,\frac{\overline{(\Delta N_i)^2}}{\overline{N}_i^2} + 1.7\,\frac{\overline{(\Delta T)^2}}{\overline{T}_i^2} + 2.1\,\frac{\overline{\Delta N_i \Delta T}}{\overline{N}_i . \overline{T}}$$

(Schulz and Brown)

$$\frac{\overline{(\Delta I_i)^2}}{I_{io}^2} = 0.64\,\frac{\overline{(\Delta N_i)^2}}{\overline{N}_i^2} + 0.64\,\frac{\overline{(\Delta U_w)^2}}{\overline{U}_w^2} + 1.28\,\frac{\overline{\Delta N_i \Delta U_w}}{\overline{N}_i . \overline{U}_w} + 0.16\,\frac{\overline{\Delta N_i \Delta T}}{\overline{N}_i . \overline{T}} + 0.16\,\frac{\overline{\Delta T \Delta U_w}}{\overline{T} . \overline{U}_w}$$

(Kulgein)

and

$$\frac{\overline{(\Delta I_i)^2}}{I_{io}^2} = 0.56\,\frac{\overline{(\Delta N_i)^2}}{\overline{N}_i^2} + 0.56\,\frac{\overline{(\Delta U_w)^2}}{\overline{U}_w^2} + 1.1\,\frac{\overline{\Delta N_i \Delta U_w}}{\overline{N}_i . \overline{U}_w} + 0.19\,\frac{\overline{\Delta N_i \Delta T}}{\overline{N}_i . \overline{T}} + 0.19\,\frac{\overline{\Delta T \Delta U_w}}{\overline{T} . \overline{U}_w} \tag{4}$$

(Clements and Smy: Thin Sheath)

In each case, the theory indicates that the fluctuations in the probe current do not really represent fluctuations in ion density unless certain conditions are fulfilled. The static probe theory of Schulz and Brown (and also of Zakharova et al. - Su and Kiel), requires that

$$\frac{\overline{(\Delta T)^2}}{\overline{T}^2} << 1 \quad \text{and} \quad \frac{\overline{\Delta N_i \Delta T}}{\overline{N}_i . \overline{T}} << 1$$

whereas the kinetic theories require that

$$\frac{\overline{(\Delta U_w)^2}}{\overline{U}_w^2} << 1 \quad \text{and} \quad \frac{\overline{\Delta N_i \Delta U_w}}{\overline{N}_i . \overline{U}_w} << 1$$

At the low pressures employed in these experiments the chemistry in the wake is far from equilibrium. In the near wake, the temperature and charge density fluctuations will be correlated, since the ionization arises in the high temperature fluid (created during passage through the near-normal portion of the bow shock around the stagnation point on the front of the sphere) and the cold which is subsequently entrained into the turbulent core of the wake is unionized. Ellington (34) has reported that the temperature and velocity fluctuations are anticorrelated in hypersonic turbulent sphere wakes. A correlation of ion density and temperature fluctuations would thus also imply that ion density and velocity fluctuations are anticorrelated.

The same work by Ellington provided estimates of $\overline{(\Delta T)^2}/\overline{T}^2$ varying from 0.1 to 0.2 and of $\overline{(\Delta U_w)^2}/\overline{U}_w^{\,2}$ varying from 0.2 to 0.35. Experimentally the value of $\overline{(\Delta I)^2}/\overline{I}^2$ is of the order of 0.5 (26). Accordingly, from the dependence of $\overline{(\Delta I)^2}/\overline{I}^2$ derived analytically from the various theories and stated in Equation 4, one is forced to admit that the temperature and velocity fluctuations in the wake could provide important contributions to the fluctuations observed in the saturated current of an ion probe. From theoretical considerations there is little hope that the correlation functions and spectra, obtained from statistical analysis on the experimentally observed ion probe currents, are simply related to the statistical characteristics of the fluctuating ion density in the wake.

Fortunately, support for the position that probe current fluctuations are dominated by ion density fluctuations is forthcoming from some experimental observations by Ghosh et al. (35) concerning the correlation between ion and electron probe currents in a turbulent plasma flow. Their measurements were done in an argon plasma jet. In the test region the jet had a Mach number of about 0.35 and a gas temperature of 1000 °Kelvin, while ambient pressure varied between 1 and 3 torr (36). These conditions are reasonably similar to those encountered in the wake of a 2.7 inch diameter sphere flown at 14,500 feet/second in

nitrogen at 7.6 torr (17). The mobilities for ion motion in argon and nitrogen are also comparable. The exact dimension of their probes was not given by the authors (35) but in previous work they described their probe as being 3 millimeters long and about 0.005 inch (0.125 millimeters) in diameter (36).

Because of the nature of the plasma source, the precaution was taken of first ensuring that extraneous correlation oscillations such as power supply ripples or oscillations did not distrub the measurements. Observations were then performed on the operation of two probes located 1.5 millimeter apart in the test region. The two probes were geometrically identical, but one was biased to collect a saturated electron current and the other to collect a saturated ion current. Very good correlation was observed between the electron current and the ion current, indicating that the turbulent plasma fluctuations affected both currents in an identical manner. Theoretically, the fluctuating saturated currents to ion and electron probes should be differently affected by temperature, plasma potential, and gas velocity effects. That instead one observes an excellent instantaneous correlation between the currents to electron, ion and double probes was taken by Ghosh et al. (35) to be a persuasive indication that the major source of the observed fluctuation in the current to an ion probe is indeed the electron density fluctuation.

These results could be very significant for the present research project. With some small reservations, it is reasonably sure that the Langmuir probe equipment is measuring electron density fluctuations. In our experiments, however, Langmuir probe measurements were only obtained in a limited range of radial distance ($0.8 \leq R/D \leq 1.2$). Ion probe measurements were obtained over a much greater region of the wake. In the following, we will accordingly pay considerable attention to a comparison of Langmuir probe and ion probe results in the range of the wake where both exist. Evidence that the two measurements represent the same phenomenon would greatly increase the value of the turbulent scale data obtained with the ion probe survey array.

3.0 ANALYSIS

3.1 General

Estimates of turbulent scale data can be obtained from the fluctuating probe signals through the use of a small extension to the basic analysis procedures previously described (14, 17).

Briefly the basic procedure is as follows: first the probe signals are digitized. Secondly, the signals are divided into segments of about 0.5 millisecond duration, an analysis number is assigned to each segment, and the center of the segment is related to the appropriate axial distance behind the sphere (14). Each segment of signal from the upstream probe of a given pair of probes is cross-correlated with the signal from the downstream probe. If x_i represents the digital trace of the signal from the upstream probe and y_k represents the trace from the downstream probe, then mathematically the cross-correlation coefficient R_j can be estimated from

$$R_j = \sum_{i=1}^{N} \frac{(x_i - \bar{x})\,(y_{i+j} - (\bar{y})_j)}{\sqrt{\overline{x^2}}\ \sqrt{(\overline{y^2})_j}}, \qquad j = 0, 1, 2 \ldots r$$

where

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \quad ; \quad (\bar{y})_j = \frac{1}{N} \sum_{i=1}^{N} y_{i+j} \tag{5}$$

and

$$\overline{x^2} = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 \quad ; \quad \overline{y^2} = \frac{1}{N-1} \sum_{i=1}^{N} (y_{i+j} - (\bar{y})_j)^2$$

The value for the cross-correlation R_j at $j = 0$ or zero lag represents an estimate for the value of the pure space correlation function R ($X = d$, $Y = 0$, $Z = 0$) for the direction parallel to the wake axis. (Here X, Y, Z represent the coordinates of a cartesian system with X parallel to the axis of the wake, and d is the distance between two probes.) With Langmuir probes, d will be measured in integer multiples of 0.5 inch

or 0.185 D, while for the ion probes, d will be equal to 0.375 inch or 0.14 D. (Here of course D refers to the diameter of a 2.7 inch diameter sphere.) It whould be noted that these values of the interprobe spacing d were not chosen randomly; in fact they are in the neighborhood of the values of the turbulent space scales indicated by previous measurements (22).

3.2 Choice of an Estimator for Turbulent Scales

The integral scale has long been employed as a measure of the average size of the eddies in a turbulent flow. The integral scale L (37) is defined by

$$L = \int_0^\infty R(X)\,dX$$

where R(X) is the space correlation coefficient. Apart from the fact that R(0) = 1, estimates for R(X) in the case of the present experiments are only available for one value of X, that of the probe separation distance d. Consequently the above definition of L cannot be applied.

Under similar experimental conditions, several authors (7, 22) have approximated the cross correlation coefficient by a straight line

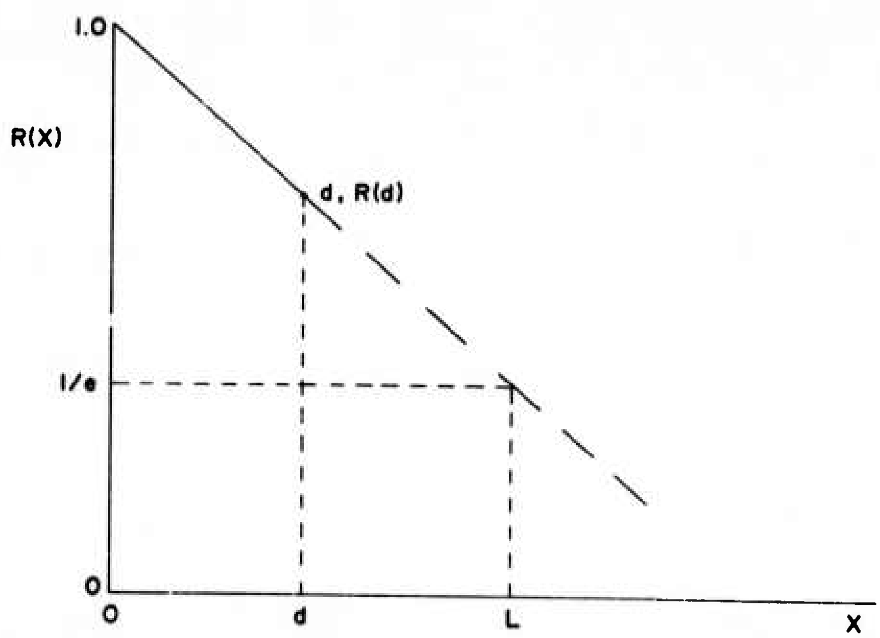

FIGURE 2

between the point $X = 0$, $R(X = 0)$ to $X = d$, $R(X = d)$ (Figure 2). The space scale was then obtained by determining the value of the abscissa X corresponding to $R(X) = (1/e)$. According to this definition

$$\frac{L}{D} = \frac{0.632}{1 - R(d)} \frac{d}{D} \tag{6a}$$

At first sight, this definition seems somewhat less than realistic. From experience (37) one expects the space correlation function to have a shape resembling the form of an exponential or gaussian curve. Thus, instead of a straight line, one could fit an exponential or gaussian curve through the two points where the space correlation function is known.

Let us assume that $\exp(-X/\Lambda)$ and $\exp(-X^2/\Lambda^2)$ represent the exponential and gaussian curves which have been fitted to the two data points $(0, 1)$ and $(d, R(d))$ on the space correlation function. Then the integral scale L_e in the exponential case is given by

$$\frac{L_e}{D} = \frac{\Lambda}{D} = \frac{1}{-\ln R(d)} \frac{d}{D} \tag{6b}$$

and the integral scale L_g in the gaussian case is given by

$$\frac{L_g}{D} = 0.886 \frac{\Lambda}{D} = \frac{0.886}{\sqrt{-\ln R(d)}} \frac{d}{D} \tag{6c}$$

The three expressions defining the integral scales L, L_e and L_g, as derived using respectively the linear, exponential and gaussian curves to fit the data points. have been calculated as a function of $R(d)$. The results are shown in Figure 3 from which one can conclude that:

(i) L_g is always smaller than L
(ii) L_e is generally larger than L
(iii) L, L_e and L_g are comparable for $0.1 \leq R(d) \leq 0.4$
(iv) $L \approx 0.5\,(L_e + L_g)$ for $0.4 \leq R(d) \leq 0.9$.

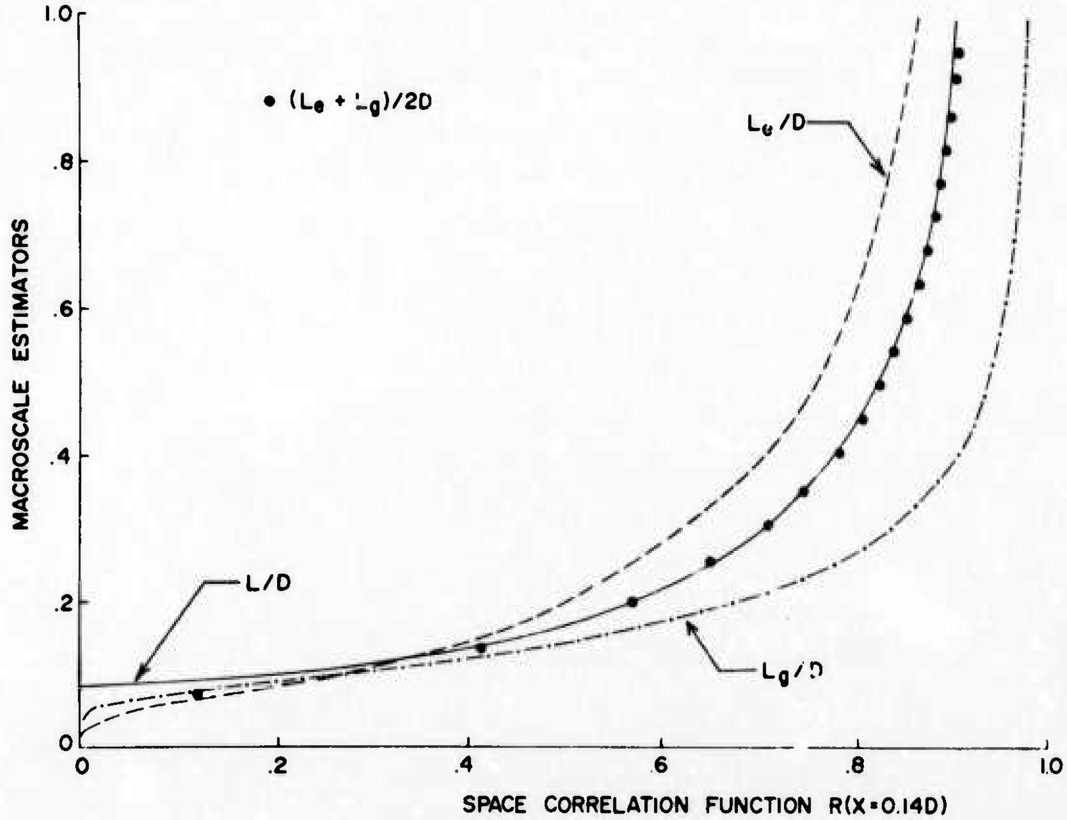

FIGURE 3

These conclusions indicate that the abscissa L/D corresponding to the value 1/e of the ordinate of a straight line drawn (0, 1) through (d, R(d)) is a valid estimator of the integral scale. The scale data presented in this report have been calculated in this manner.

3.3 The Effect of Turbulence Intermittency on Scale Estimates

In the preceding sections we have described how the probe signals have been divided into segments of about 0.5 millisecond duration, and how such signal segments are cross-correlated to produce data which lead to individual estimates of turbulent scales. In dealing with large amounts of recorded signals such as those produced with an ion probe survey array, it is necessary to resort wherever possible to automated data reduction processes. At the same time one must not lose sight of the fundamental nature of the phenomenon being measured. In the present case we are trying to measure the distribution of charge density integral

scale sizes which characterize the turbulent fluid in the wake. However, particularly in the near wake and depending strongly on its distance from the wake axis, a probe will see a succession of laminar and turbulent expanses of fluid as the wake is swept over it. Consequently, the probe signals will be intermittent with segments of fluctuations separated by quiescent or quiet periods (19). As the turbulent core of the wake grows and envelopes the surrounding laminar flow, the total wake assumes an increasingly turbulent nature. Thus the signal from a given probe will usually show first a laminar behavior, followed by a region of mixed laminar and turbulent appearance, and finally it will be predominantly turbulent and fluctuating (19). It would be difficult to design and incorporate an efficient general mechanism into the data reduction programs which would be able to characterize the signal in a given segment as being either laminar or turbulent in nature. The present programs treat all 0.5 millisecond segments of signal in the same manner, which means that segments of laminar signal could also produce estimates of space scale sizes. It is thus important to provide editing procedures that will minimize any tendency for scale estimates from laminar signals to distort the overall results.

In practice, the problem differs depending on the radial location of a probe with respect to the wake axis. At a radial distance of about one diameter, the signal tends to be predominantly turbulent from about 20 or 30 diameters in the near wake. In this case, the problem does not arise, since all the scale estimates are obtained from predominantly turbulent segments of signals. At radial distances of about two diameters, any signal detectable above the zero level is from turbulent fluid and, since no analysis is performed in the absence of signal, the problem is again mainly avoided. However, there is some difficulty with radial distances around 1.5 diameters where predominantly laminar signals frequently extend out to axial distances of about 200 diameters.

Although the data reduction processes which lead to estimates of scale sizes do not contain a built-in mechanism for differentiating between (what we shall call for convenience) "laminar scales" and "tur-

bulent scales", other means could be employed. The main effect of a laminar segment in a pair of signals which have been correlated is to produce a scale estimate which is numerically large. One can thus print out a list of the space scale estimates for a given pair of probes and pick out those which appear large. A correlation of these large space scale events can then be carried out with the intermittency data already derived for the ion probe survey array data (19), and any scales originating from low intermittency or laminar signals can be rejected or at least flagged. We shall return to this possibility later.

It is possible by some simple experimental and analytical arguments to derive some criteria for rejecting laminar scales. For example, it is physically impossible for the size of the space scale obtained from the near wake to exceed ten times the projectile diameter. In fact the diameter of the energy-carrying eddies cannot exceed the diameter of the wake. Only at 1000 diameters behind the projectile does the schlieren diameter of the wake approach ten times the body diameter (38), and in the near wake the charge density radius of the wake is considerably smaller than the schlieren radius (17). Thus, for a certain axial distance X/D, if one obtains a space scale which is larger than the wake diameter at the same position, this is a strong indication that the scale estimate was produced from the signal of a probe immersed in a laminar fluid flow.

The signal from an ion probe immersed in laminar wake flow is characterized by very slow variations, somewhat similar in form to a cosine function. Let W_1 and W_2 represent two laminar signals of slightly different frequency that are slightly out of phase with respect to one another. Then

$$W_1 = \cos 2\pi f\,(t+t_o)$$

and

$$W_2 = \text{COS } 2\pi f_1 t$$

where

$$f_1 = f + \Delta f$$

The cross-correlation coefficient between W_1 and W_2 is given by

$$\frac{\overline{W_1(t-\tau)\ W_2(t)}}{W_1(\text{rms})\ W_2(\text{rms})} = \frac{\text{SIN } 2\pi\Delta f\ T}{2\pi\Delta f\ T} \cdot \text{COS } 2\pi f(\tau - t_o) \qquad (7)$$

where T is the duration of the signal segments.

This cross-correlation coefficient is shown in Figure 4. The curve has a maximum for a value of the lag $\tau = t_o$, the magnitude of the phase shift between the two signals. The cross-correlation function at zero lag has a maximom value of COS $2\pi f\ t_o$ which is obtained when the two frequencies of W_1 and W_2 are identical ($\Delta f = 0$). Furthermore, one sees that the value of the cross-correlation coefficient at zero lag contains information concerning the apparent basic frequency of the correlated signals. For a given value of t_o, the closer the value of the coefficient at the origin tends to unity, the lower the apparent frequency of the signal or, equivalently, the more laminar the signal appears.

The higher the value of the correlation coefficient at zero lag, the higher the value of the indicated space scale. It is therefore of interest to attempt to estimate the possible magnitude of the laminar scale. First we must estimate the frequency f and the time t_o. The latter represents the time shift between the two correlated signals, and is approximately the time required by the local wake velocity to transport a turbulent pattern from one probe of a pair to the other (14): $t_o = d/U_c$, where d is the interprobe distance and U_c the local convection velocity. In the case of the ion probe pairs, d = 0.14D = 0.0315 ft.

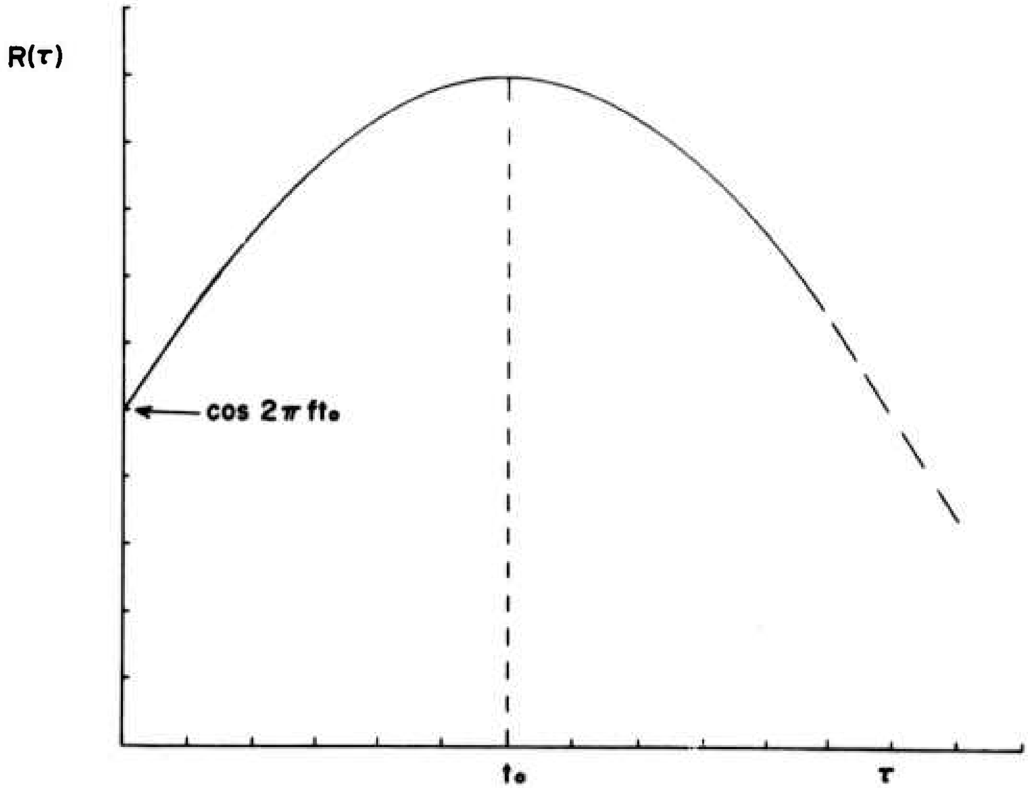

FIGURE 4

The maximum value of U_c observed by a pair of probes at 7.6 torr, is only greater than 2000 feet/second about 1% of the time and no values greater than 3000 feet/second were obtained (14). In consequence, the value of t_o cannot be smaller than 10 microseconds and it is rarely smaller than 15 microseconds.

The value of the frequency f is difficult to estimate, but some idea can be gained from the following considerations. Since the basic length of a segment of signal used in the analysis is 0.5 millisecond, the minimum frequency that one could detect would be of the order of 2 kilohertz. It is useless to worry about frequencies much lower especially since, to the eye, a signal containing low frequency components of about 4 or 5 kilohertz appears relatively laminar. Table I lists a number of values of t_o and f and indicates the sort of laminar scale sizes which they would predict, based on Equations 6a and 7.

TABLE I

POSSIBLE SIZES OF LAMINAR SCALES

f(kilohertz)	t_o(microseconds)	L/D
1	5	179
2	5	44
3	5	19
4	5	11
3	10	5
4	10	2.8
4	15	1.3
4	20	0.7

Table I indicates quite conclusively that space scales derived from segments of laminar signal containing slow or low frequency variations will tend to be quite large. One possible technique of eliminating laminar scales consists of automatically dropping all estimates of space scales that exceed a certain value. This criterion would not necessarily eliminate all laminar scales, but it would reject those which were most likely to distort the average result.

4.0 RESULTS

4.1 General

The scale estimates obtained during these experiments were distributed with respect to the axial and radial distance coordinates in a manner completely analagous to the velocity estimates obtained in our study of wake velocity (14). As a result the procedures followed in grouping the data points to determine the mean behavior of the turbulent integral scales are very similar to those previously defined for wake velocity. Usually, the statictical properties were calculated for groups of data points defined by axial bands about 60 or 100 diameters wide. In addition, because of interest in the homogeneity of the turbulence, the data were also grouped into radial bands of a width of 0.3 diameter.

To justify the manner in which the scale estimates are presented, it is necessary to anticipate the results somewhat. Briefly, the scale results obtained with the ion probe array indicated an independence of the scale size with radial position in the wake. Accordingly it was decided to represent the mean profile of the radial distribution of space scale estimates by a straight line parallel to the abscissa representing the radial coordinate. The obvious technique is to calculate the position of this straight line from the arithmetic mean of all the space scale estimates in the radial distribution. However, a test of the goodness-of-fit shows that the space scales are not distributed according to the Gaussian law. Instead, the distribution of the estimates was asymmetric, similar to the Rayleigh distribution.

Accordingly, a representation of the behavior of the space scales by only the arithmetic mean seems inadequate. However, one must also keep in mind that the space scales were estimated by a linear two-point extrapolation or interpolation technique. As a result, most of the graphs give the arithmetic mean of the data and at the same time indicate the distribution of the data points by means of the 25th, 50th and 75th percentile curves.

While scale estimates exceeding one sphere diameter are shown on the graphs, they have been ignored when computing the arithmetic mean and the quartile curves. This is certainly justified at small X/D values. At values of X/D > 1000 the space scale estimates show signs of increasing in magnitude and large scale estimates should probably be included. In the present data these large values are so infrequent that no larger error is made by ignoring them.

4.2 Scale Estimates from Langmuir Probe Observations

All of the Langmuir probe scale estimates were obtained by firing 2.7 inch diameter spheres at 14,500 feet/second in 10 torr air atmospheres. A presentation of the results of scale estimates from observations with Langmuir probe axial arrays on two consecutive rounds is given in Figure 5a. In both cases, the probes were constructed from 2.3 millimeter lengths of 0.0005 inch diameter platinum wire. While the probes were biased at + 9 volts with respect to ground, adjustments to the plasma potential are believed to have occurred so that the actual potential difference between the probes and the plasma was about + 1 volt (18). The data shown in Figure 5a were deduced from the signals of the second and third probes in the Langmuir array.

The fluctuating electron density scale data obtained with the Langmuir probes (Figure 5a) extend from an axial distance of about 40 diameters to more than 2000 diameters behind the spherical projectile. In both cases the probes saw an initial short run of laminar signal. The scale estimates derived from these laminar signal segments at axial distances of about 50 diameters are large compared to unity. Subsequent to X/D = 100, the signals are continuously fluctuating and with few exceptions most of the scale estimates are less than unity. These 10 torr air results are consistent with expectations based on the data obtained in nitrogen at 7.6 and 20 torr (19). In particular, the measurements of the intermittency structure of hypersonic 2.7 inch diameter sphere wakes (19) indicate that the wake mean intermittency radius exceeds one diameter at axial distances greater than 100 diameters. Since for both rounds represented in Figure 5a, the probes were located

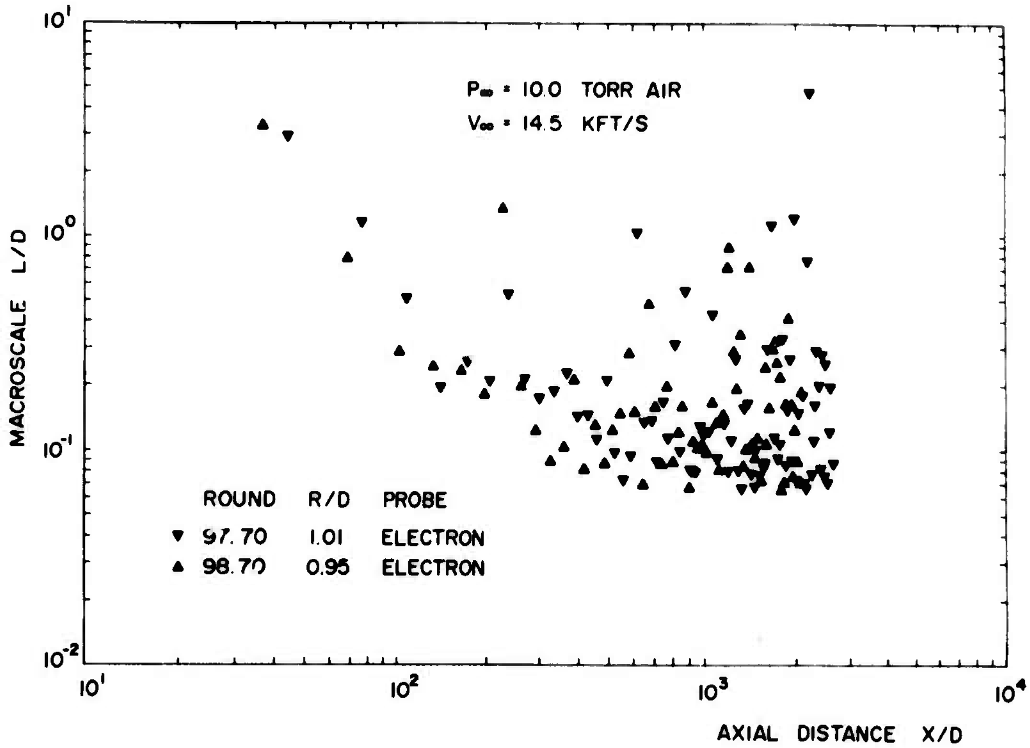

FIGURE 5a

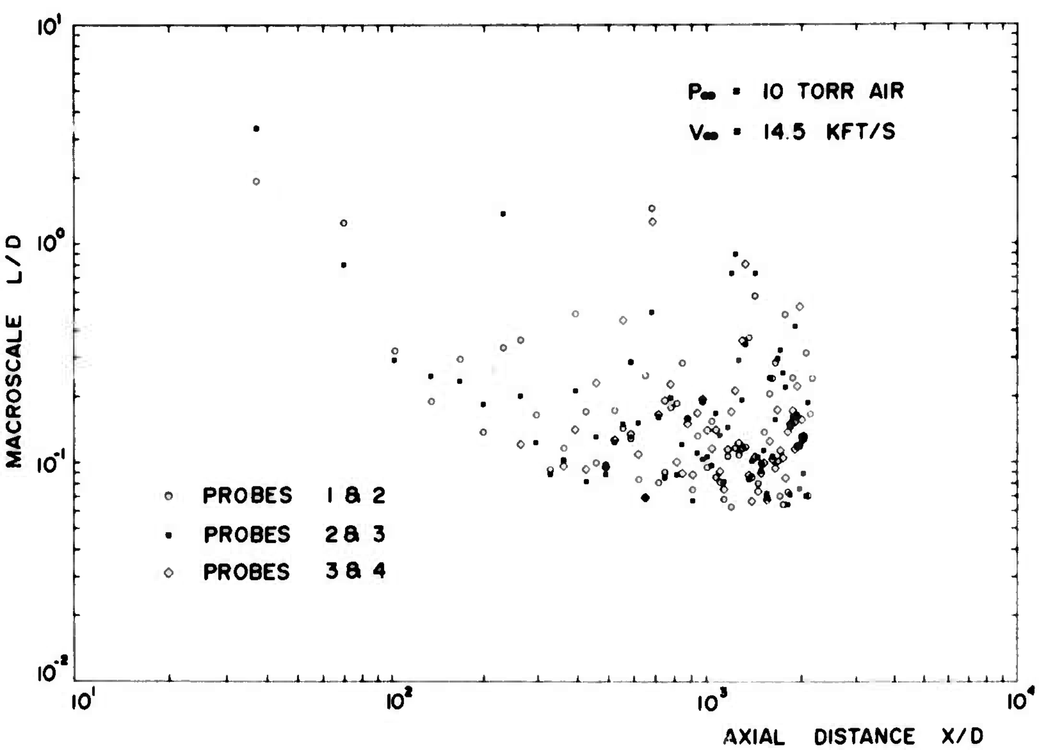

FIGURE 5b

within one body diameter of the wake axis — 1.01 D for Round 97.70 and 0.95 D for Round 98.70 — it is to be expected that the probes would be immersed in predominantly turbulent fluid subsequent to X/D = 100.

Figure 5b shows turbulent scale estimates deduced from three different possible pairs of adjacent probes in a 4-probe axial array. The indices indicate which pair of probes was used to obtain a particular data point: for example probes 1 and 2 represent the pair formed by the first and second probes in the array, starting with the upstream probe. The results are not sensitive to the choice of a particular pair of probes.

Figure 6 presents Langmuir probe or electron density scale data from 6 different rounds. For data obtained when the Langmuir probe array was employed with the Mode 1 scheme (18) of different biases on different probes, the signals from the pair of probes having the highest biases were used to deduce the scale estimates. All of the Langmuir probe scale estimates were obtained in the radial distance band extending from 0.8 to 1.1 diameters.

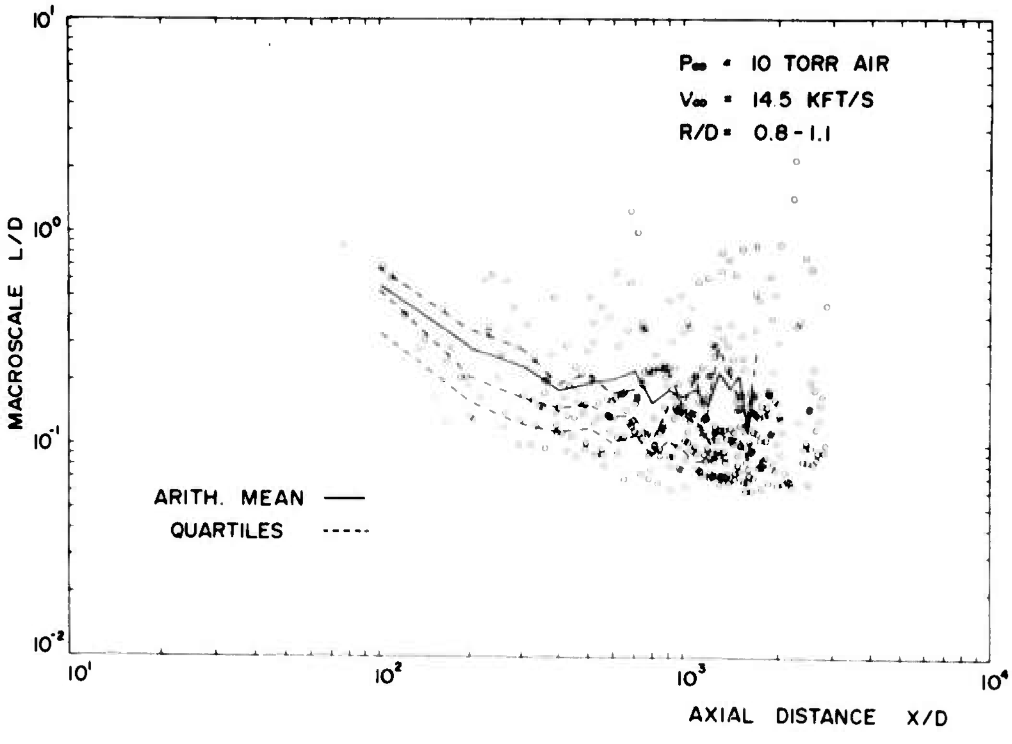

FIGURE 6

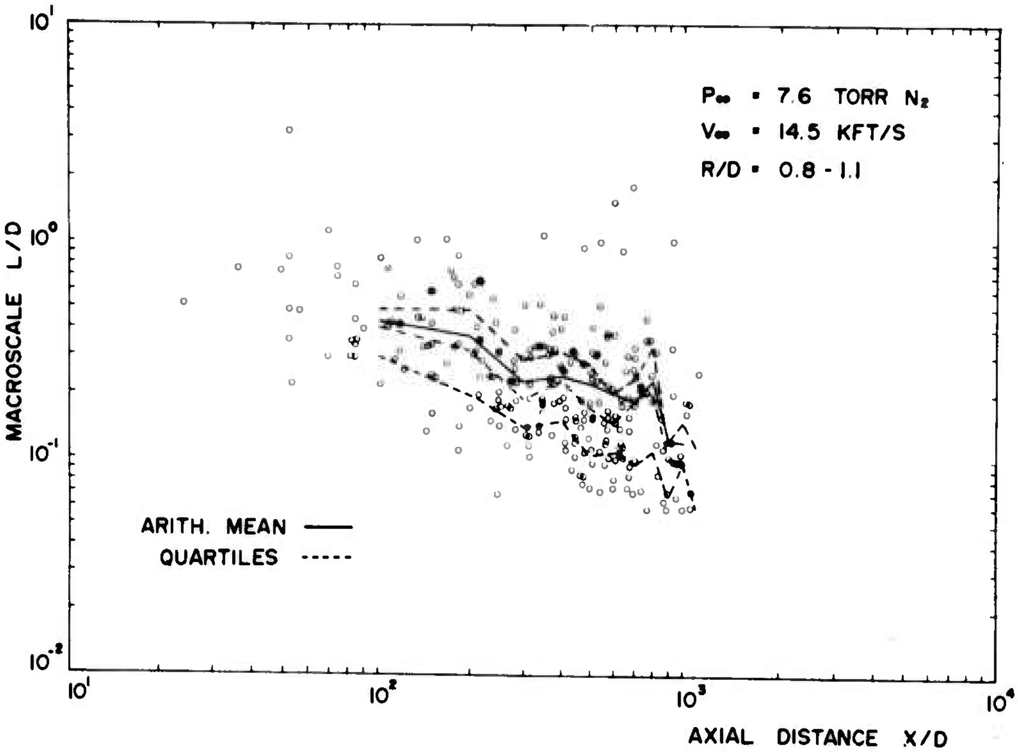

FIGURE 7a

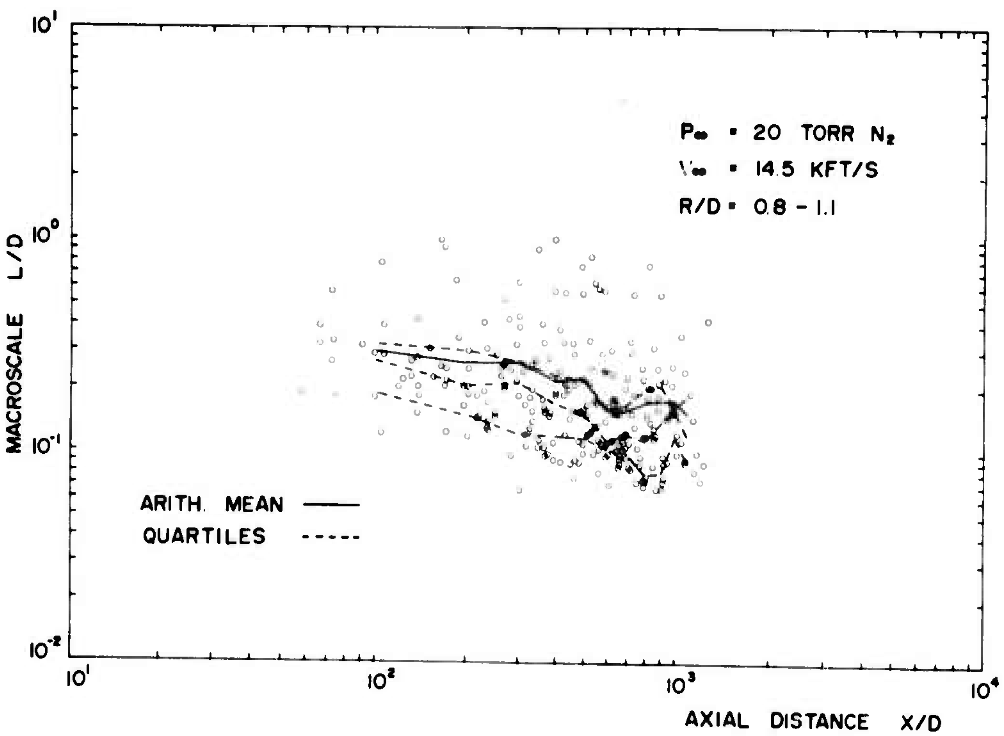

FIGURE 7b

The solid line and the dashed lines in Figure 6 represent respectively the arithmetic mean and the 25th, 50th and 75th percentile limits computed by clumping the data into axial bands defined by the combination 100-100-100 (14). In order, these three numbers represent respectively the axial position of the first axial band, the distance between the successive axial bands, and finally the width of the bands (all distances are measured in sphere diameters). The arithmetic mean of the data decreases over the first few hundred diameters, thereafter remaining roughly constant at about 0.2 diameter. At around 2000 diameters there is some indication that the mean scale is beginning to increase. It will be noted that the arithmetic mean falls between the 50th and 75th percentiles, indicating the skewed distribution of scale estimates.

4.3 Comparison of Scale Estimates from Electron Probes and Ion Probes

Almost all the scale estimates obtained with the Langmuir probe array were restricted to the radial distance band $0.8 \leq R \leq 1.1$. The ion probe survey results were not so restricted. Accordingly, in order to effect a comparison of the electron probe and ion probe results, the ion probe estimates were also grouped into a number of radial distance bands. Figure 7 (a and b) shows the ion probe scale estimates in the radial distance bands $0.8 \leq R/D \leq 1.1$ obtained from the results at 7.6 and 20 torr in nitrogen. The arithmetic mean curve and the quartile curves previously described are also shown. Scale estimates exceeding one diameter were ignored when computing these curves.

The comparisons of the arithmetic mean curves and the median (50th percentile) curves for the turbulent scale estimates obtained with the electron probes at 10 torr and with the ion probes at 7.6 and 20 torr in the same radial distance band ($0.8 \leq R/D \leq 1.1$) are shown in Figure 8 (a and b). There is no strong reason to expect any dependence of the scale behavior on the difference in the atmospheric composition at the

same pressure. The number of data points in the three cases are comparable, although the distribution of the points with axial distance are slightly different insofar as there are almost no electron scale estimates for X/D < 100, while there are no ion scale estimates for X/D > 1000. The axial distance bands into which the scale estimates have been clumped are centered at X/D = 100, 200, 300, etc. and the values at these points are connected by segments of straight lines.

A comparison of the arithmetic mean scale results in Figure 8a indicates that the electron scale and the ion scale estimates are very similar, particularly if less weight is given to the electron scale value at X/D = 100 because of the small number of data points for X/D < 100 in this case. (The difference between the 7.6 and 20 torr nitrogen data at small axial distances is a real effect to which we will return later.) The median curves (Figure 8b) display a trend similar to that shown by the arithmetic means, although the coincidence of the curves is not as good, mainly because of the departure of the results at 7.6 torr from those at other pressures.

Figure 8 is considered to provide strong evidence that the turbulent scales deduced from the fluctuating currents seen by ion probes are equivalent to the turbulent scales deduced from the fluctuating currents detected with Langmuir probes, under the range conditions employed. Since the fluctuations in the plasma potential are believed to have been small in these experiments, the Langmuir probe current scales presented here are probably closely representative of electron density fluctuation scales. Consequently the behavior of the more extensive ion probe scale results can be used with considerable confidence as a valid indication of the behavior of the electron density fluctuation scales in the wakes studied in the present research.

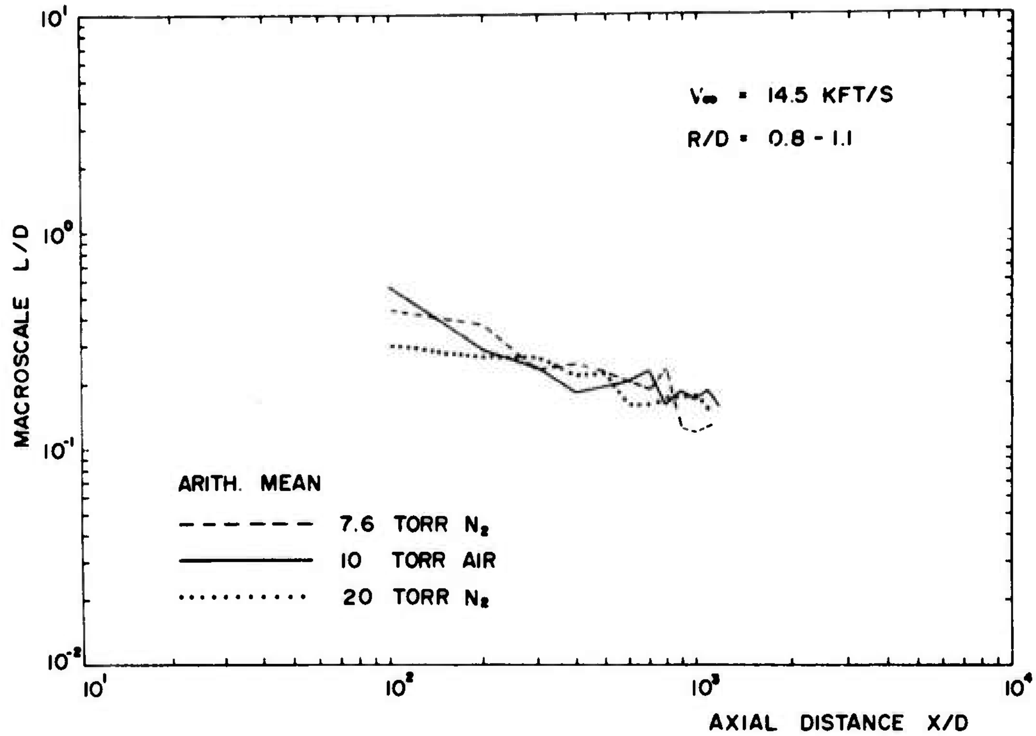

FIGURE 8a

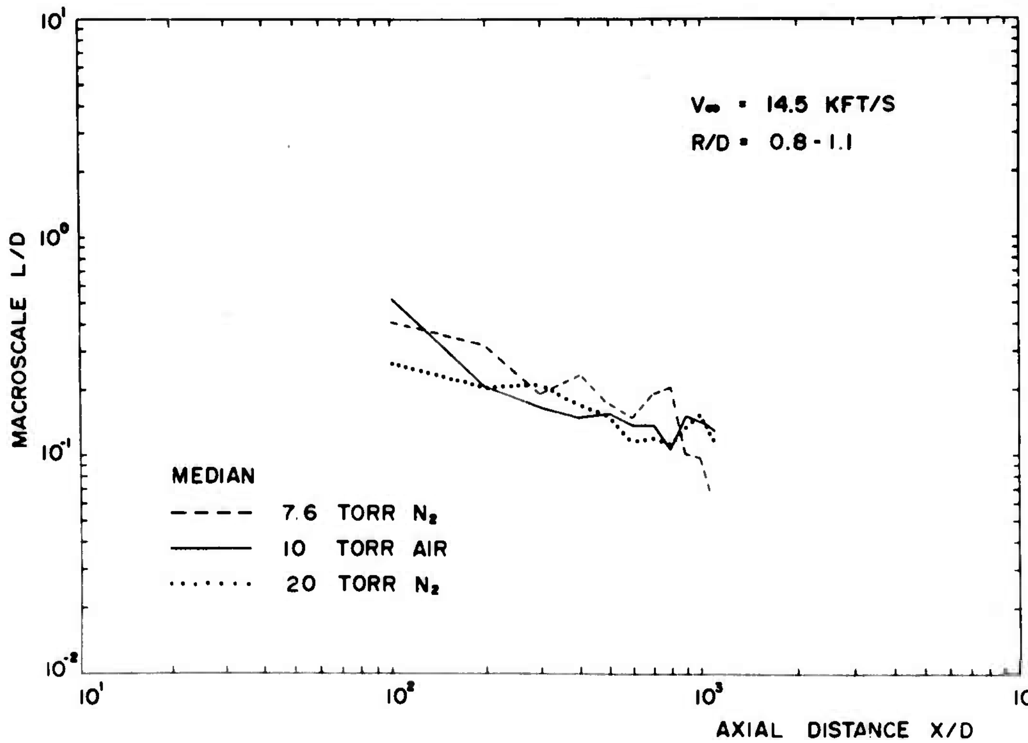

FIGURE 8b

4.4 Scale Estimates from Ion Probe Array Observations

Figures 9 and 10 present additional results for ion probe scale estimates similar to those already presented in Figure 7. This figure showed the ion scale estimates in the radial distance band extending from 0.8 to 1.1 diameters, while Figure 9 and Figure 10 are concerned with the radial distance bands from 1.4 to 1.7 and from 1.9 to 2.2 diameters respectively. The arithmetic mean and the quartile curves have been computed as previously described; scale estimates which exceed one diameter are plotted but were not included in calculating statistical averages.

A comparison of the 7.6 torr ion probe scale results given in Figures 7a, 9a and 10a shows that there is an usually large number of numerically large scale estimates in the radial band for $1.4 \leq R/D \leq 1.7$ Examination of the probe signals indicates that this appearance of large ($L/D > 1$) scales is directly related to the intermittency structure of the probe signals from which the scale estimates were deduced (10). The 7.6 torr ion probe signals have a large concentration of low intermittency factor signal segments in the radial band from 1.4 to 1.7 diameters. At small radial distances, the signals are mainly turbulent, the intermittency factor is high and fewer large laminar scales are obtained. At large radial distances (1.9 to 2.2 diameters) only the turbulent eddies are sufficiently ionized to be detected, and the number of large scales deduced from the signals is smaller than that deduced from the intermediate radial distance band (1.4 to 1.7 diameters). The discarding of scale estimates larger than one diameter appears to eliminate practically all the scales contributed by signal segments with a low intermittency factor. There may still be some residual effect due to intermittency on the 7.6 torr scale data, but because of the different nature of the turbulence at 20 torr (19) there is probably very little effect on the scales obtained at the latter pressure. Thus the scale estimate results presented in this report are believed to approach those which would be obtained from fully conditioned turbulent signals.

The next step is to compare the trends of the scale estimates in the three different radial bands. This is done for the arithmetic mean scales in Figure 11a for the 7.6 torr results and in Figure 11b for the 20 torr results. Our conclusion is that at both pressures the trend of the data is independent of radial distance. The turbulence in the wakes at 7.6 torr and 20 torr appears to be homogeneous, in agreement with the previous conclusions of the MIT group (2, 5).

Accordingly, it is reasonable to lump all the scale data at one pressure together, independently of radial distance. Figure 12 shows typical radial profiles of scale estimates obtained at the two pressures by clumping data points in the axial distance band from 390 to 450 diameters (Scale estimates exceeding one diameter have been suppressed.) The straight lines passing through the data parallel to the abscissa axis indicate the position of the arithmetic mean of the data points. The average trends of the ion probe scale estimates with increase of axial distance are shown in Figure 13, where both the arithmetic means and the quartiles are presented. Here the results have again been obtained by clumping data in axial bands of 100 diameters in width.

Figure 14 compares the scale results obtained at 7.6 torr and those obtained at 20 torr; the arithmetic mean curves are shown in Figure 14a and the median (50th percentile) curves in Figure 14b. Over the axial distance range extending from 100 to 1000 diameters, the average scale size for the 20 torr data decreases from a value of about 0.3 diameter to about 0.15 diameter. The average scale size for the 7.6 torr data is about 1.5 times the size of the 20 torr scale results in the axial interval between 100 and about 400 diameters. Thereafter the scale sizes seem to be the same at both pressures; the increasing fluctuations at an axial distance of 1000 diameters are probably mainly due to decreasing numbers of data points. The difference between the character of the ion probe signals at 7.6 torr and 20 torr has previously been qualitatively noted by the authors in their study of the structure of turbulent hypersonic sphere wakes (19).

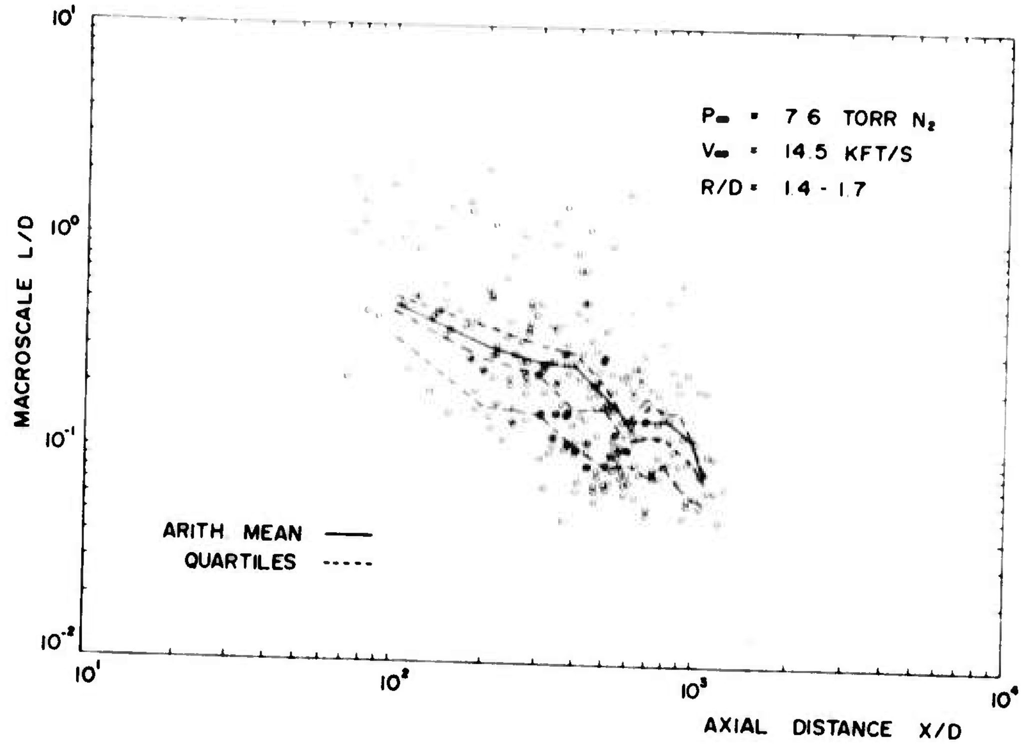

FIGURE 9a

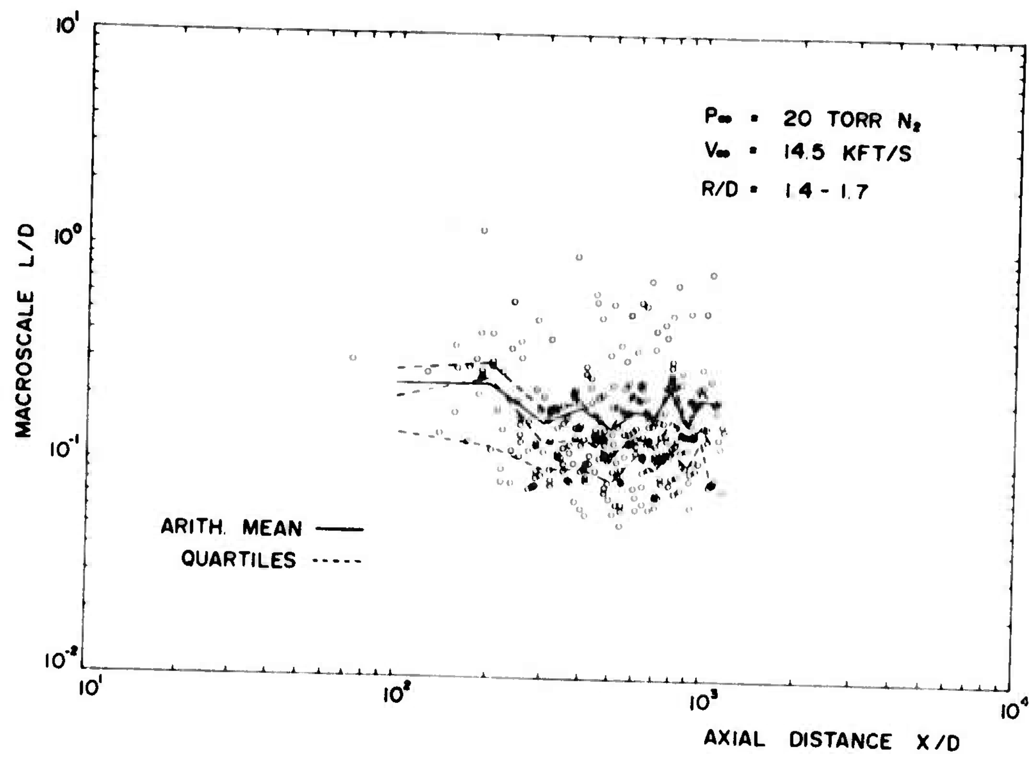

FIGURE 9b

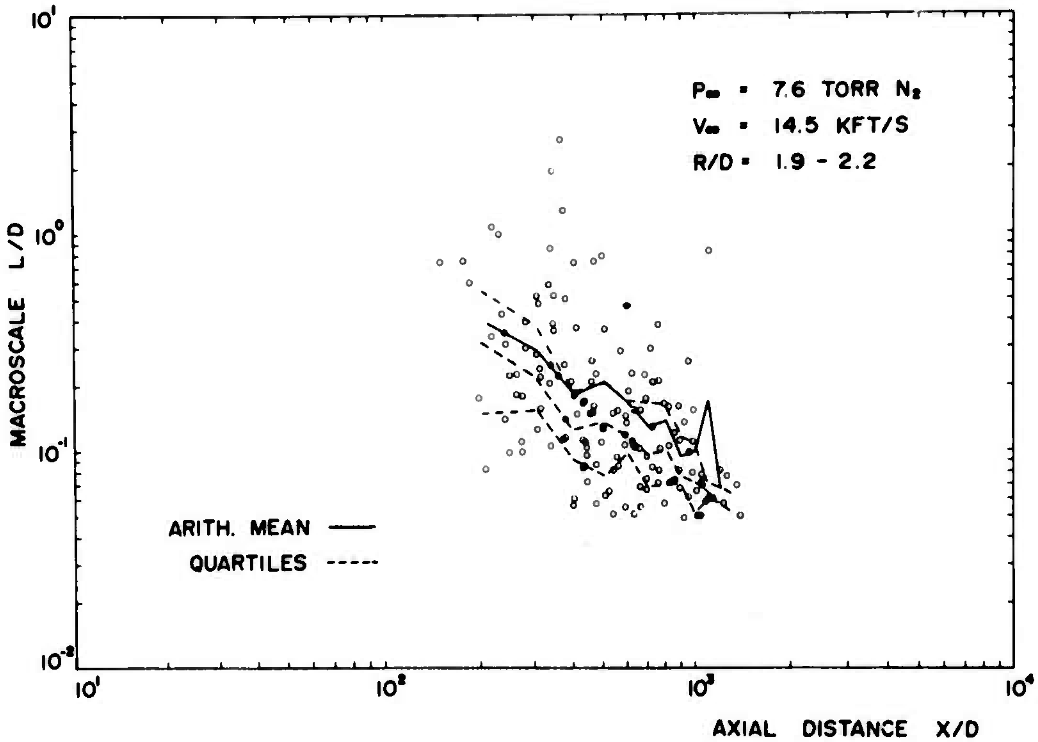

FIGURE 10a

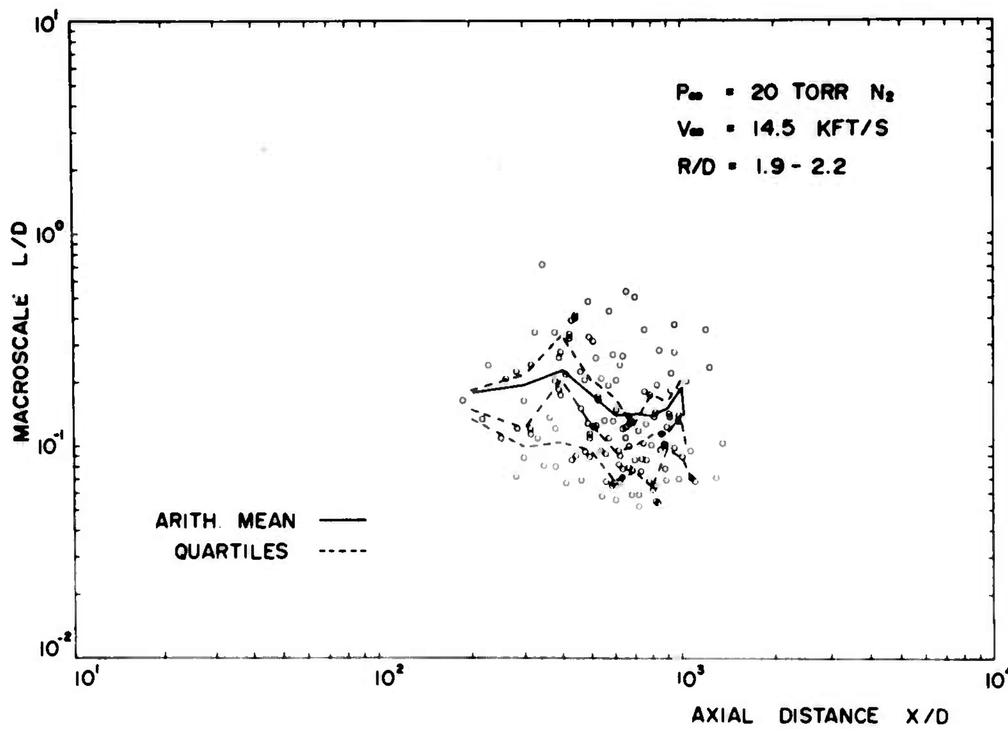

FIGURE 10b

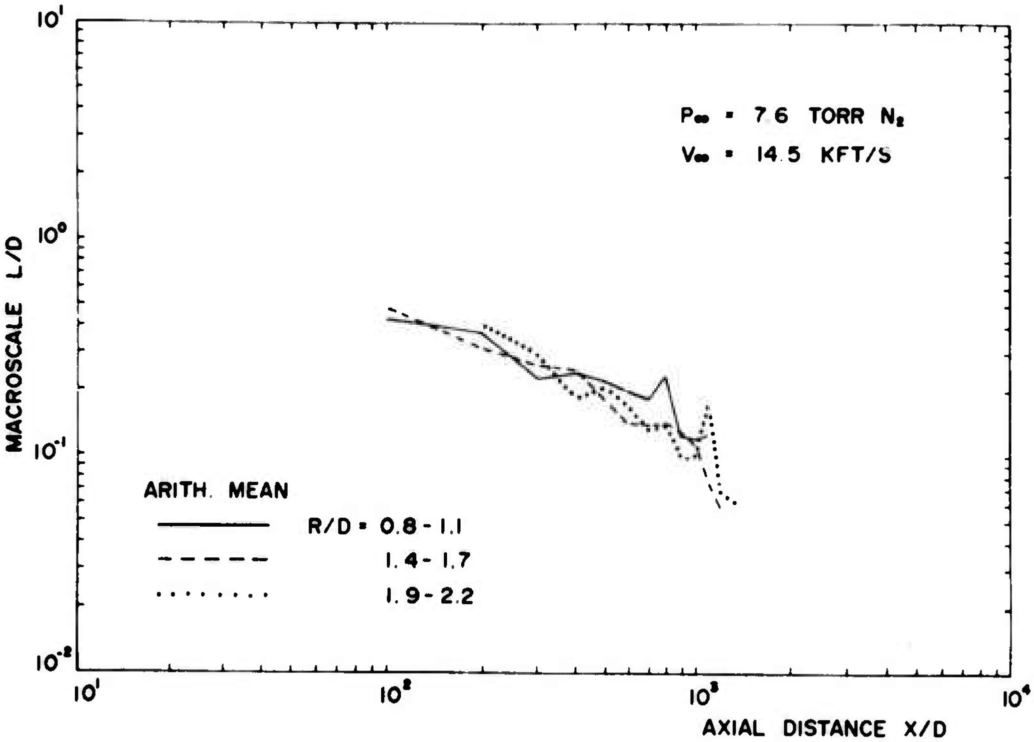

FIGURE 11a

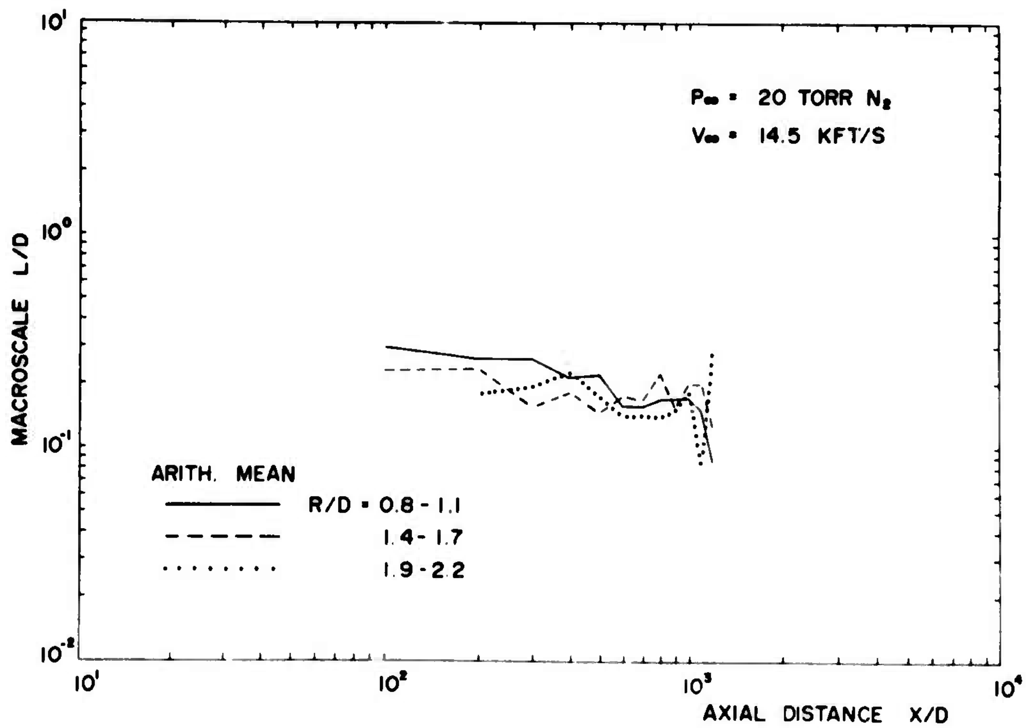

FIGURE 11b

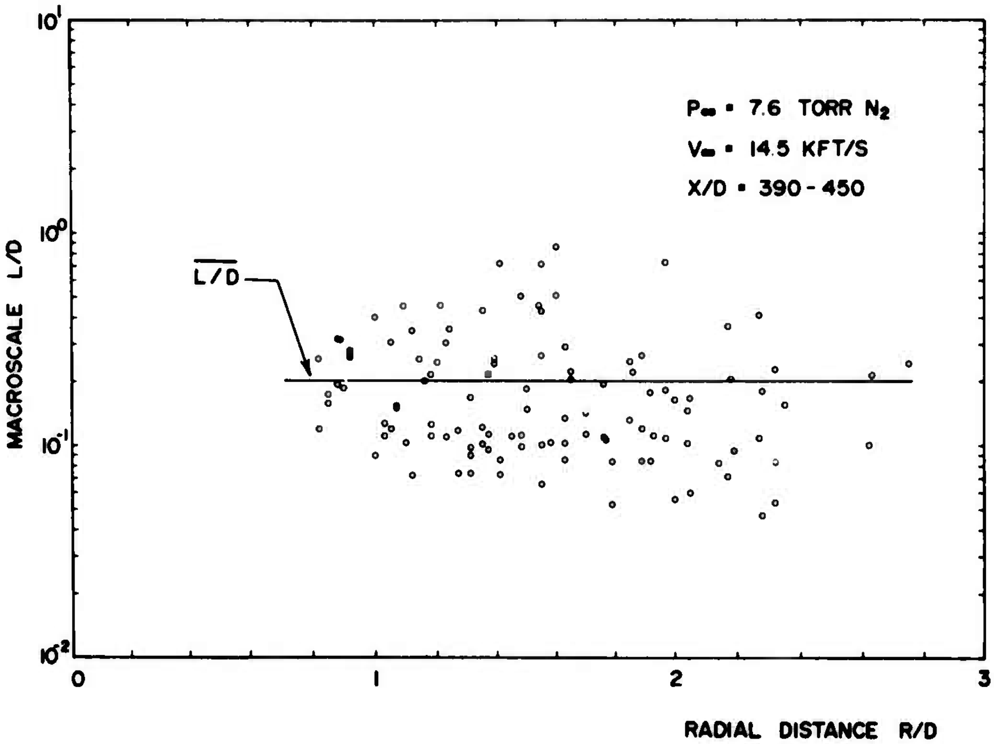

FIGURE 12a

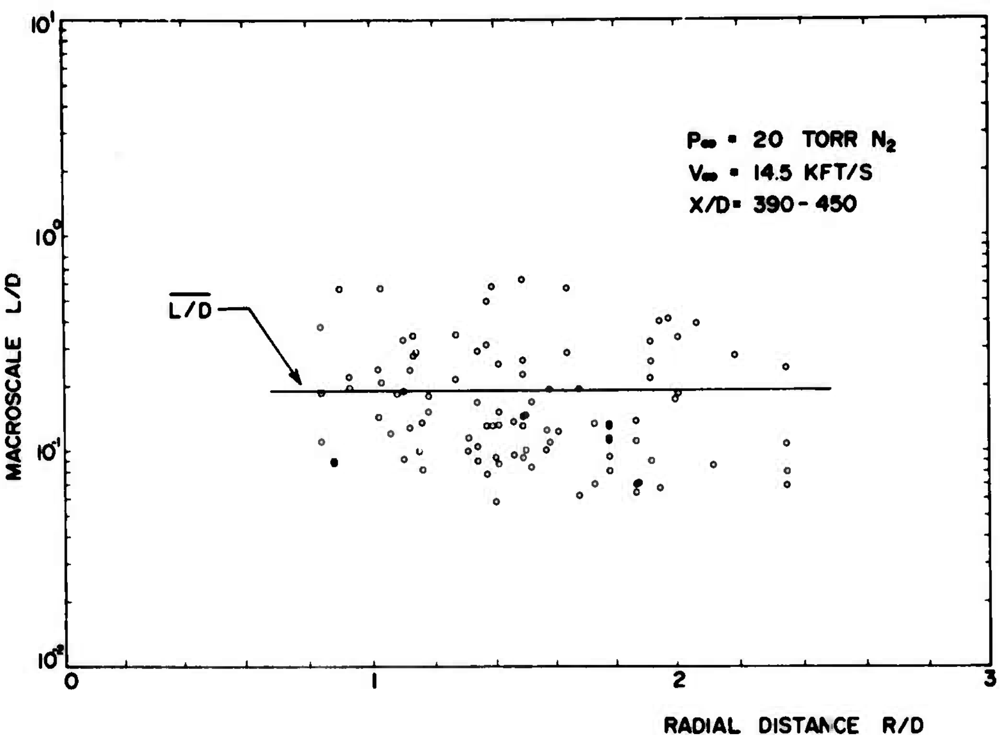

FIGURE 12b

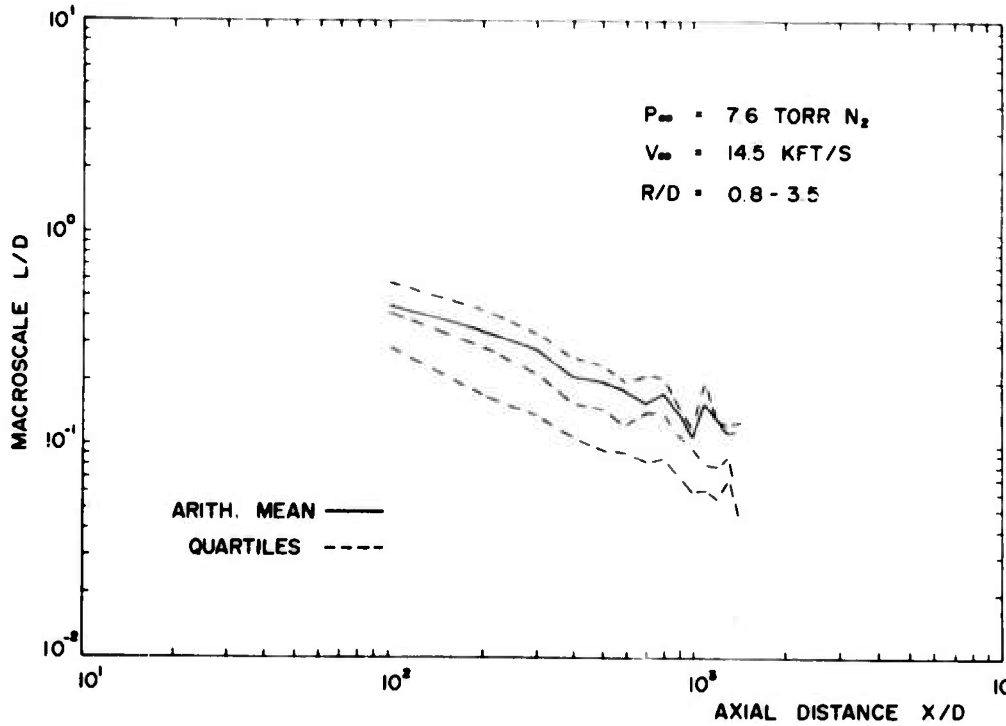

FIGURE 13a

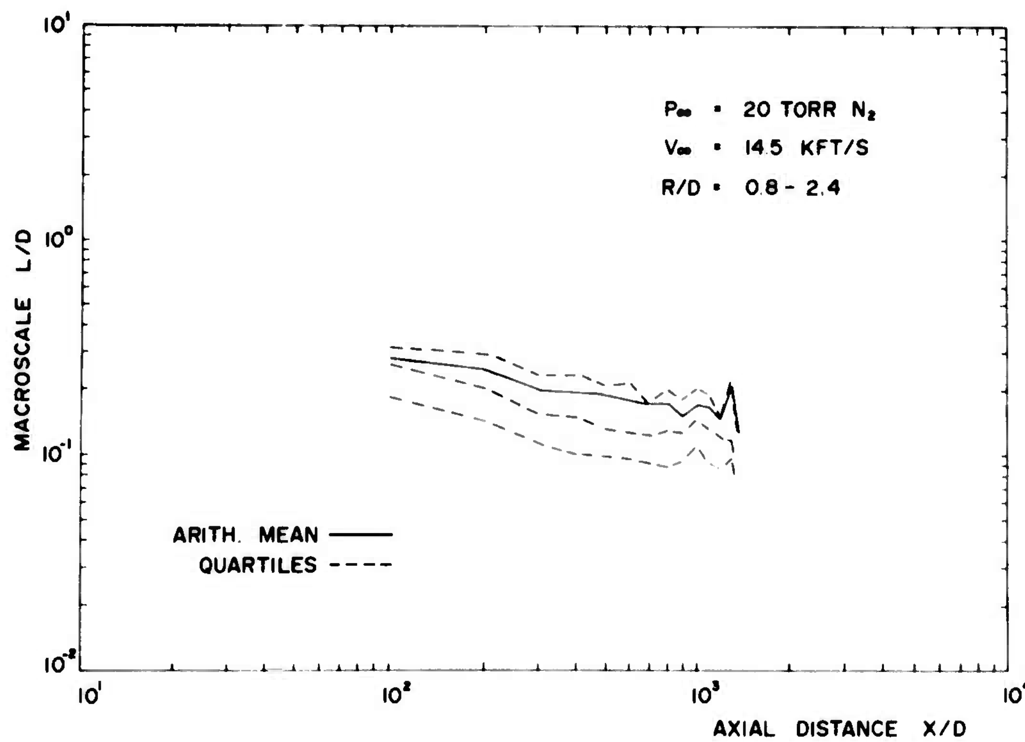

FIGURE 13b

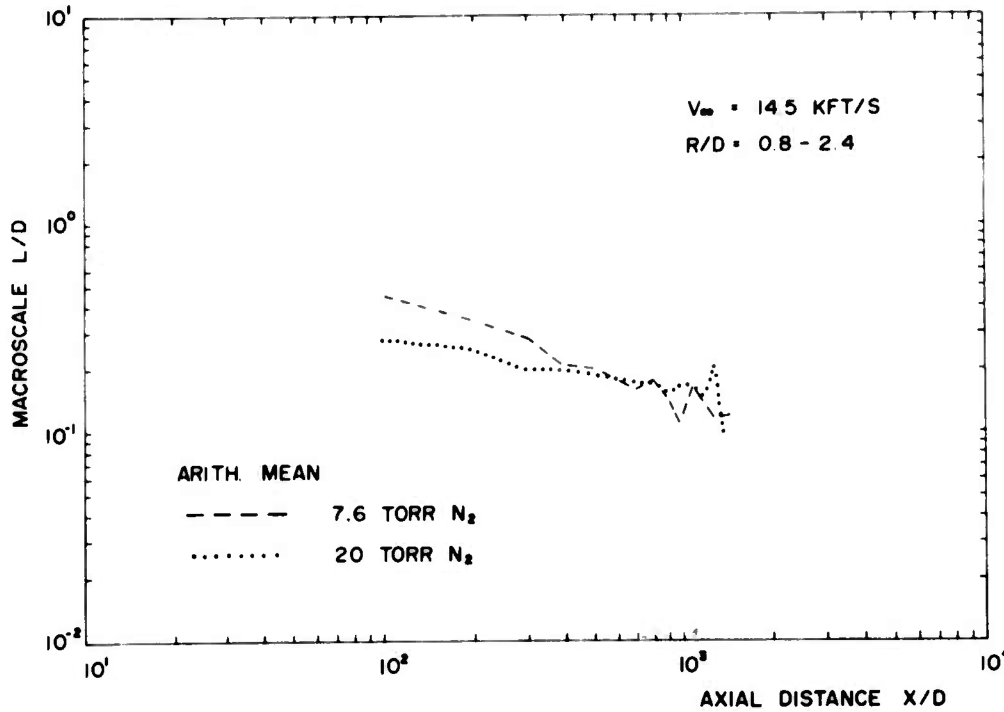

FIGURE 14a

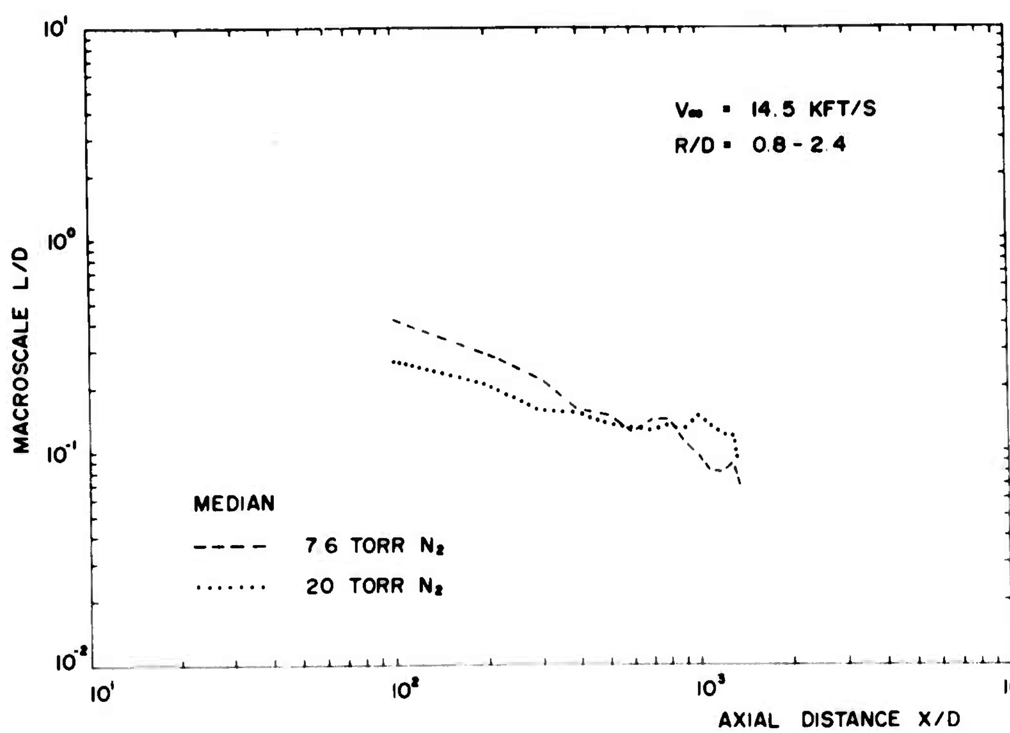

FIGURE 14b

4.5 Statistical Accuracy of the Mean Ion Probe Scales

Figure 15 presents the arithmetic mean scale results obtained at 7.6 torr and 20 torr again, but in a slightly different format. The results are based upon the same scale estimates used to construct Figure 14a, but here the axial bands are defined by the combination 60-30-60 (14). The first axial band is thus centered at X/D = 60, the distance between successive bands is 30 diameters and the width of the bands is 60 diameters. The axial behavior of the mean macroscales is portrayed in Figure 15 with slightly more resolution but with some loss in at some cost to statistical precision.

To estimate the statistical precision of the mean estimates, the law of Student was used:

$$\left[(\bar{x} - \frac{st_{n;\alpha/2}}{\sqrt{N}} \leq \mu_x \leq \bar{x} + \frac{st_{n;\alpha/2}}{\sqrt{N}})\right], \qquad n = N-1 \qquad (8)$$

where

$\bar{x}$ is the mean value

s is the standard deviation

$t_{n;\alpha/2}$ is the percentile of the law of Student

N is the number of points, and

μ_x is the expected value.

While this law is only strictly true for a gaussian distribution, one may employ it as a first approximation in other statistical distributions.

The vertical lines in Figure 15 show the 90 percent confidence limits. The horizontal lines indicate an uncertainty in axial position of about 30 diameters, corresponding to the basic length of a correlated segment of probe signal. Obviously the differences observed at axial distances smaller than 400 diameters between the mean scale behavior at 7.6 torr ($P_\infty D$ = 20 torr-in) and at 20 torr ($P_\infty D$ = 54 torr-in) are statistically significant.

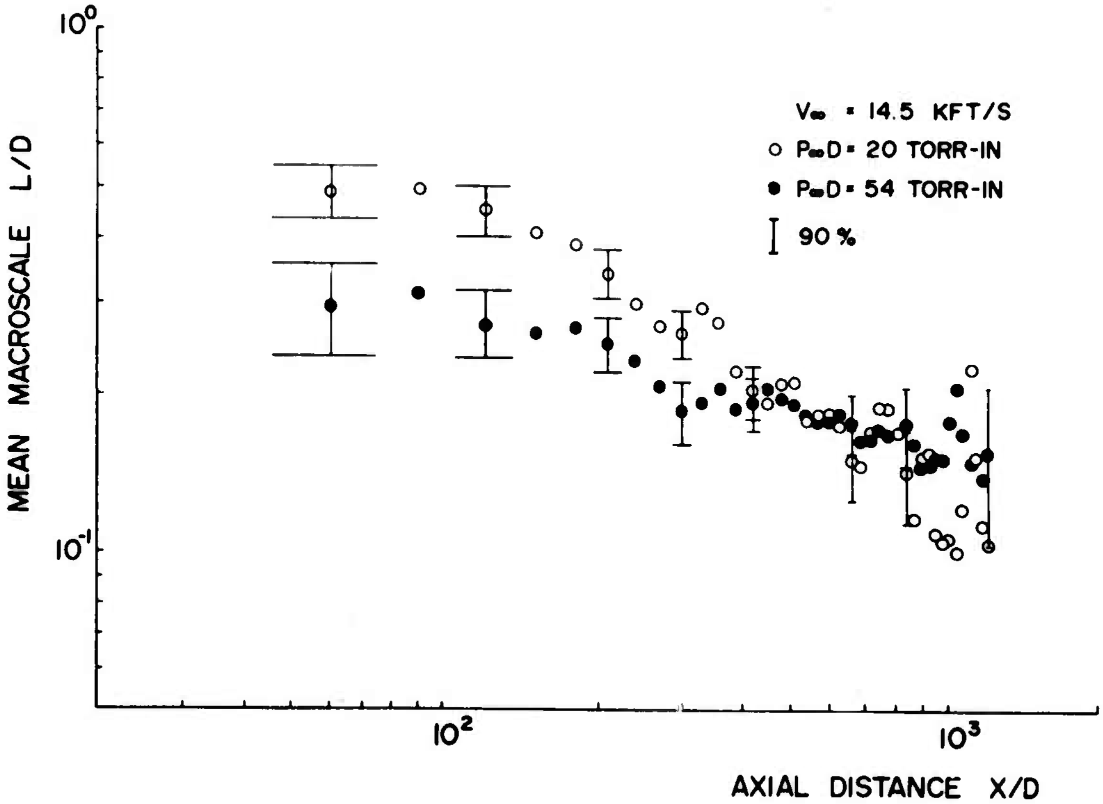

FIGURE 15

In Figure 15 we have deliberately reverted to the ballistic range practice of employing the ambient-pressure-times-projectile-diameter parameter $P_\infty D$. For 2.7 inch diameter spheres, $P_\infty D$ has the value of 20 torr-inches at 7.6 torr and 54 torr-inches at 20 torr. This parameter is extremely useful when comparing results obtained at different range facilities.

5.0 DISCUSSION

Most of the previous graphs have dealt with wake macroscale estimates obtained from ion probe signals. Up to the present time, very little credence has been placed in the value of scale results obtained from fluctuating signals detected by ion probes immersed in hypersonic wakes, probably as a result of the difficulty of interpreting probe current fluctuations indicated by analysis based on various theoretical expressions for the ion probe current. The value of the present ion

probe scale data has been greatly increased by the experimental demonstration that for the conditions employed in the DREV ranges, the ion probe scale estimates are very similar to Langmuir electron probe scale estimates. These Langmuir probes are believed to have operated under the conditions required to ensure that the probe current fluctuations were dominated by electron density fluctuations. Accordingly the ion probe macroscale behavior is thought to be closely indicative of the behavior of electron density fluctuation scales in the wake.

Some of the more interesting results obtained are the indicated homogeneity of the turbulence in the wake, the differences between the 7.6 torr and 20 torr macroscale results at axial distances less than 400 diameters, and finally the indications of the initial decrease in the mean scale size with increase of axial distance. In this latter instance similar behavior has been calculated theoretically by Schapker (39).

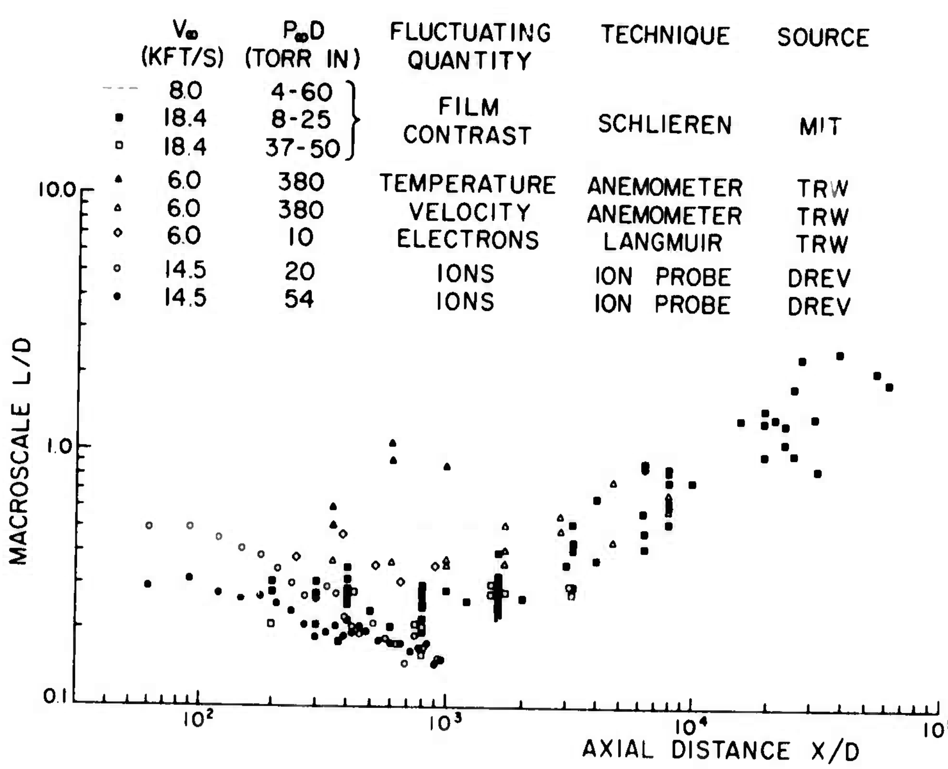

FIGURE 16

Figure 16 presents a comparison of the mean DREV ion probe macroscale results with the scale measurements obtained in most of the previous work discussed in the Introduction. The circles represent the DREV results, the triangles and the diamond-shaped symbols are results obtained at TRW, while the squares and the thin dashed line represent MIT observations. Because of the large number of symbols employed in Figure 16, the results at first glance seem somewhat confusing; the confusion would probably be even greater if the low speed MIT schlieren film contrast data were represented by points rather than by a dashed line. Nevertheless a certain number of tendencies in the data can be discerned.

The TRW fluctuating temperature scale results have considerably higher values than the others and no further attention will be paid to them. Only the DREV ion probe macroscales indicate a clear tendency for the average scale size to first decrease with axial distance in the region from 100 to 1000 diameters and to be dependent on the parameter $P_\infty D$. Although there is no indication of the fact in Figure 16, the DREV Langmuir probe scale data do exhibit the beginning of a trend for the scale size to increase with axial distance in the neighborhood of $1000 < X/D < 2000$. There is thus considerable reason to believe the behavior with axial distance, indicated by all the macroscale data taken together, where the mean scale size first decreases and then increases with increasing axial distance.

Over the axial distance range between 100 and 1000 diameters, where there is a considerable overlap of different kinds of data, a few fairly well defined differences exist among the various results. The 8000 feet/second MIT schlieren film contrast scales are larger than the 6000 feet/second TRW velocity and electron scales; in turn the TRW macroscales are larger than the 18,000 feet/second film contrast scales and these are larger on the average than the DREV ion probe scales. That the mean macroscale is dependent on velicity is indicated by the differences between low and high speed film contrast data as well as the difference between TRW electron and DREV ion scales.

6.0 CONCLUSION

The behavior of macroscales associated with the charge density fluctuations in the turbulent wakes of hypersonic spheres has been investigated using both Langmuir electron probes and continuum electrostatic ion probes. With cylindrical Langmuir probes operated under favourable conditions, theoretical analysis indicates that the statistics of the probe current fluctuations are representative of the statistics of the electron density fluctuations. Similar analysis for continuum ion probes indicates however that the statistics of the current fluctuations depend not only on the ion density fluctuations but may also be strongly influenced by correlation functions involving temperature or velocity fluctuations.

Nevertheless, in the present experiments involving probe measurements in the wakes of 2.7 inch diameter spheres flown at 14,500 feet/second in 10 torr air and in 7.6 and 20 torr nitrogen atmospheres, macroscale measurements with ion probes appear to be equivalent to macroscale measurements from Langmuir probes. This result greatly increases the credence of the ion probe macroscale data which, in fact, are considerably more extensive than those obtained with Langmuir probes.

The mean macroscales obtained at DREV with ion probes decrease in magnitude as axial distance increases between 100 and 1000 diameters; the Langmuir probe results which extend to slightly larger axial distances indicate the mean scale begins to increase again between 1000 and 2000 diameters. For axial distances less than 400 diameters, the ion probe results also indicate a significant dependence of the mean macroscale on the paramter $P_\infty D$. When compared with previous macroscale results obtained in sphere wakes by a variety of techniques, the DREV measurements appear to fit into a general pattern in which the mean macroscale first decreases and then increases with increasing axial distance. This same comparison indicates that the mean macroscale behavior is dependent on sphere velocity.

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